VOLUME 1, No. 6 (1956)

NUCLEAR PHYSICS

MONTHLY JOURNAL DEVOTED TO THE EXPERIMENTAL AND THEORETICAL STUDY OF ATOMIC NUCLEI, NUCLEAR FIELDS AND THE FUNDAMENTAL ASPECTS OF COSMIC RADIATION

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NUCLEAR PHYSICS is published in monthly issues. The subscription price of a volume of 700 pages is fl 57.00; 114s; $ 15.00. Postfree.

SUBSCRIPTIONS should be sent to the publishers, NORTH-HOLLAND PUBLISHING COMPANY, P.O.Box 103, Amsterdam or to any subscription-agent. Subscriptions for U.S.A. are to be sent exclusively to INTERSCIENCE PUBLISHERS, INC., 250 Fifth Avenue, New York 1, N.Y.

Printed in the Netherlands.

Nuclear Physics, North-Holland Publishing Co., Amsterdam, **1** **(1956)**

THE INTERACTION OF NEUTRON AND ALPHA-PARTICLE

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Received 26 January 1956

Abstract: The phenomenological neutron-alpha interaction is determined from the phase shifts in the scattering of neutrons on alpha particles, over the neutron energy range 0—10 MeV. From the notion of the alpha particle, acting as a single unpolarised unit in the scattering, comes the assumption that the interaction is mainly of the ordinary velocity-independent type, which may be easily subjected to phenomenological analysis. A new general analysis relates useful phase parameters to parameters defining interaction potential. For a consistent solution, on the basis of an ordinary potential with spin orbit coupling, which would give the main phase data, for the S, P and D waves, one requires the interpretation of the S phases in terms of an attractive potential which has one bound state, rather than by a predominantly repulsive interaction. On this interpretation the 'bound state' is not observed because the wave function for the corresponding $(1s)^5$ state will vanish identically by the exclusion principle. Using the group resonance formalism the ordinary potential so obtained can be related to fundamental force parameters in a consistent way, because the neutron-alpha interaction is indeed seen to have only a secondary velocity-dependence for these force parameters. The fundamental force-parameters are close to those required by low energy nucleon-nucleon data; they give the correct alpha binding energy, and have an exchange character which is non-saturating without a repulsive core.

1. Introduction

The problem to be discussed is the determination of the phenomenological interaction potential of neutron and alpha particle, as evidenced in neutron-alpha scattering over the neutron energy range 0-10 MeV. The symbol He^5 will be used below merely as a concise reference to aspects of the neutron-alpha interaction. Having determined the He^5 phenomenological interaction it shall be related to fundamental forces through the theory for the He^5 wave equation provided by the group resonance formalism.

The relevant experimental data of Seagrave [1]), and Huber and Baldinger [2]), interpreted in terms of S, $P_{3/2}$, $P_{1/2}$ phases mainly, show considerable divergences in the actual phase values, especially for the $P_{1/2}$ phases. There is agreement, however, about the signs and general trends with energy. Using the Wigner-Eisenbud resonance

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reaction formalism, Dodder and Gammel [3]) deduced (n, α) phases from the more accurate (p, α) scattering data. These phases agree very well with the experimental (n, α) phases of Seagrave. The present calculations shall assume the (n, α) phases due to Dodder and Gammel, and Fig. 1 shows a plot of the main phases. Due to the divergences of the sources, it is to be doubted whether the small D

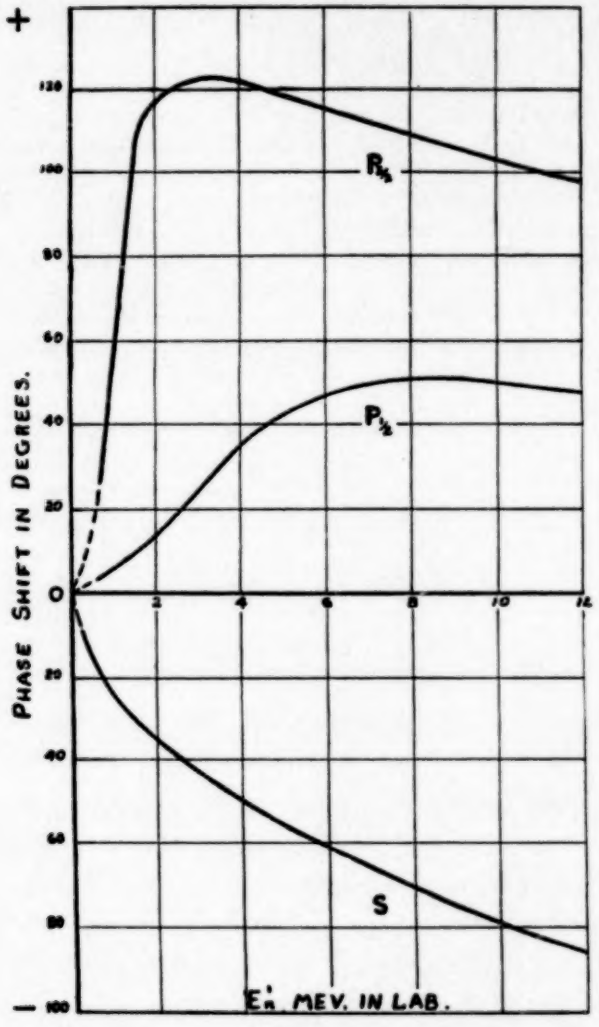

Fig. 1.
(n,α) phases [3]).

phases can be insisted upon. It shall be assumed that the main requirement is that the D wave phases shall be small in magnitude.

The phases immediately indicate strong He^5 interaction, with spin orbit coupling of the type corresponding to inverted doublets.

The P wave phases correspond most naturally to effective He^5 interaction potentials which are predominantly attractive. The fact that the S phases do not permit of any obvious comparison with a reasonable definition of the mean P phases, but are in fact of the opposite sign, immediately indicates that the S phases may need a special interpretation. Call the S phases plotted in Fig. 1, η_0. The same cross-sections would have been obtained, of course, with S phases $= \pi+\eta_0$. Taking this and the fact that no bound S state is observed, one may note two possible alternative interpretations of the S wave data:

(a) An S wave He^5 interaction which is predominantly repulsive. In view of the P wave data, this means an S wave interaction with a considerable repulsive core. Such an interaction would be the most natural interpretation for two interacting particles or groups which are truly indivisible in the scattering and associated phenomena, and the repulsive core would then represent the saturated core of the $(1s)^4$ configuration of He^4.

(b) An S wave He^5 interaction which is predominantly attractive and corresponds to a phase of $\pi+\eta_0$, and has one 1s bound state. The phase parameters for the S wave can be shown to allow of a binding energy of 11.0 MeV. This interpretation would be in conflict with the absence of an observed bound S state, if the He^5 problem really were a two-group problem also at such large negative energies, as it shall be suggested below that it is for low positive scattering energies. It is, however, consistent with the absence of an observed bound state for He^5, considered as a five-body problem at such large negative energies, because a $(1s)^4$ state is saturated and thus the wave function for a $(1s)^5$ bound state will vanish identically by the exclusion principle. Therefore the „bound state" is in fact unobservable, because it has a zero probability of being occupied. This possibility corresponds to a zero energy phase shift of π, and is thus related to the discussion of excluded bound states and zero energy phase by Swan [4]). The present discussion differs from the latter in suggesting that there are here two possible and distinct alternatives in principle, which would lead to quite different fundamental forces:

A predominantly repulsive S wave He^5 interaction, (a), corresponds to fundamental forces of the type which gives absolute saturation; and the predominantly attractive S wave He^5 interaction, (b), to fundamental forces of a type which does not give absolute

saturation without the agency of say a repulsive core in the elementary interaction.

A possible theoretical treatment of the He^5 interaction as a scattering problem, say for neutron energy 0—10 MeV, is to derive the He^5 wave equation in terms of fundamental force parameters, by using the group resonance formalism [5]). This can only be done readily, and in a useful form, for a relatively simple elementary force law, and assuming that the alpha-particle does not polarise in the scattering. The latter is probably a good assumption over the above energy range for the well-bound alpha particle, which has an excitation energy probably of the order of 20 MeV. One must decide on the proper low energy phenomenological fundamental forces. These are of course incompletely known. In line with a general trend for the He^5 problem, central forces of a general exchange character and a Gaussian radial dependence shall be assumed. The precise effect of tensor forces is still undecided, but may undoubtedly appear in two ways:

(1) On the He^5 S wave interaction, and its natural concomitant, the He^4 binding, they have a mainly indirect effect [6,7]) through the selection of a consistent set of central exchange forces. The small direct effect may be described as that of an equivalent central force, and the resultant central force is here suggested to be sensibly the same for the He^5 interaction as for He^4 binding.

(2) In the He^5 spin orbit coupling: the largest estimate of the contribution of tensor forces to this coupling is provided by the work of Feingold [8]), and is still considerably below the magnitude suggested by Dodder and Gammel [3]). In view of the fact that Feingold's calculation treated the virtual $P_{1/2}$, $P_{3/2}$ levels as bound, and that a small effective range of the wave function was seen to be required, Feingold's estimate may even be an overestimate.

Thus the tensor force shall be neglected, at least explicitly, and the spin orbit coupling considered in two ways:

(1) As a purely phenomenological He^5 interaction, which could include tensor force effects;

(2) As due to a fundamental nucleon-nucleon spin orbit coupling.

On the basis of these assumptions about forces, and the He^5 problem, one can deduce the He^6 wave equation, and it shows two types of He^5 interaction:

(i) an ordinary force with a Gaussian well shape, and with spin orbit coupling;

(ii) an integral interaction term which makes the He^5 interaction velocity-dependent. Apart from direct dependence of the interaction on neutron energy, one has another type of velocity-dependence in the dependence of the kernel of the integral term on l, the He^5 orbital quantum number, called, say, l-dependence.

It is conceivable that this l-dependence may be relevant to the large differences between the mean P wave and the S wave data, seen in Fig. 1. Especially in the case of interpretation (a), above, of the S wave data by a predominantly repulsive He^5 interaction, this l-dependence may enable one to have the predominantly attractive He^5 interaction required by both P waves. Since part of the considerable repulsive core required in interpretation (a) of the S wave data, may, furthermore, be furnished by a repulsive core in the elementary forces, it is useful to indicate some effects which such a feature would have, and which is otherwise neglected in the calculations below. It may make such an interpretation of the data possible, based on interpretation (a) of the S wave data, and requiring appreciable velocity-dependence of the He^5 interaction, if the elementary force repulsive core is sufficiently strong. Furthermore, a strong repulsive core in the elementary force law, may in itself directly increase the integral interaction: Over the domain of the integration variable in (ii), say r', where r' is small, both the kernel of the integral and the wave function is small as $(r')^{l+1}$, and this where the interactions are strongest. If the interactions have a core of a sufficiently strong and singular kind, one may nevertheless have an important contribution from the domain of small r', whereas this is less likely for a Gaussian force law.

Because it leads to specific correlations of the particles involved, a strong repulsive core in the elementary force will render the proper calculations of the He^5 wave equation much more complicated than in Section 2, for a Gaussian force law. Unless the elementary force core is very singular, or otherwise predominates over the attractive part of the elementary force, the motion of the constituents of the alpha particle makes it possible that the contribution of the core of the elementary force to the repulsive core of the He^5 interaction will be of a radius somewhat smaller than the overall radius of five elementary cores in closest packing. If the elementary core is not very strong, it may be that the conclusions arrived at below, neglecting the elementary core, may need only minor adjustments with this core.

As is well known, there are many arbitrary features in relating phases, known over a limited energy range, to interaction, especially when the latter could be velocity-dependent as for He^5, or when the system has bound states [9]). This arbitrariness may be resolved by appeal to other guiding principles. One could derive the He^5 wave equation, as suggested above, and, since it is very complicated, one could solve it numerically for a likely set of fundamental force parameters [10,11]). This limits one to relatively simple force laws, and depends directly on the validity of the specific use of the group-resonance formalism, say in taking only two unexcited groups. Even then, some arbitrariness remains in, for example, the relative importance of terms (i) and (ii) in the interaction, and for given force laws, in deciding the best compromise fit for the three partial waves. In any case, a complete family of exchange parameters will give one particular numerical form for the wave equation. Until the consistency problem of nuclear forces is solved, it is uncertain whether a set of parameters which is successful in this way, gives the correct solution; the more so because it may be difficult to test the interaction so derived in related problems, if it contains an appreciable integral term. It seems desirable to consider an alternative procedure which would complement the above approach.

The alternative procedure which leads to a phenomenological approach, is offered here by considering the one participant in the scattering, the alpha particle. The latter is a well bound particle, particularly for its mass range, and it is not easily excited. Thus one may expect the alpha-particle to act sensibly as one unpolarised unit, at low positive scattering energies, or an essentially two-body interaction which has no apparent reason to be velocity-dependent. Even in the derivation of the wave equation of He^5, using the group resonance formalism, which naturally introduces the constituents of the alpha particle initially, one has that in the ordinary interaction, (i), the alpha constituents play a completely implicit role in merely determining the probability density of the alpha structure versus position. The integral interaction term, (ii), shows a clearly explicit dependence on constituents of the alpha group, and exchange operations on them. For an alpha particle acting as though a single unit, one may expect the He^5 interaction to be mainly of type (i), with an integral interaction which may first of all be neglected. Then the He^5 interaction is velocity-independent and of the ordinary type. Thus in the present paper one looks for a velocity-independent

phenomenological potential with spin orbit coupling to cover the three important partial waves. In this form the analysis of the phase data can be independent of a specific derivation of the wave equation, and the interaction found may later be readily tested in related problems, such as the discussion of $(4n+1)$ nuclei. Since one finds 8 significant phase parameters from an analysis of the phase data for the three main partial waves, one may expect specific information about the well shapes for one ordinary potential with a spin-orbit coupling potential. This and considerations of simplicity will help to remove the remaining arbitrariness in the calculation of the interaction from the phases. The divergences of the sources of phase data may limit the detailed significance of the complete interaction function so found, at the present time. Thus it seems desirable at present to establish a connection to fundamental forces at the same time, by using the group resonance formalism, as indicated above, to provide a theory for the ordinary interaction. This means finding a best compromise fit using ordinary forces of Gaussian well shape.

This phenomenological approach on the basis of no velocity-dependence, does not yield a satisfactory solution using interpretation (a) of the S wave data (predominantly repulsive S wave He^5 interaction); but one does find a consistent solution on interpretation (b) of the S wave data (attractive S wave He^5 interaction). It has already been mentioned that a velocity-dependent solution of type (a) may be possible if the elementary interaction has a strong repulsive core. This possibility is not pursued further below.

Section 2 then sketches the derivation of the He^5 wave equation for a Gaussian fundamental force law. Section 3 introduces a general analysis which relates parameters defining the phases to parameters defining the interaction potential, which has been very useful in the phenomenological analysis. Section 4 attacks the phenomenological problem on the basis of interpretation (a) of the S wave phases, and velocity-independence. Section 5 gives the phase parameters derived from a least square analysis of the phase data, and ends with a purely phenomenological analysis of the $P_{3/2}$ interaction, showing a preference for a well which resembles a Gaussian in shape. Section 6 gives the best fit of the data on the basis of velocity-independence and interpretation (b) of the S data, using square wells and Gaussian wells. The latter is then related to fundamental force parameters, using the theory for the ordinary interaction derived in Section 2, with the He^4 binding energy as a subsidiary condition. Section 6

concludes with an estimate of the neglected integral interaction term, using the derived fundamental parameters.

2. The He^5 Wave Equation

Here a fundamental theory for the He^5 interaction shall be given. The analysis is based on the group resonance formalism [5]), and uses two unpolarised groups: the alpha particle and the neutron.

The neutrons shall be denoted as particles 1, 2, 3 and the protons as 4, 5. The small effect of the Coulomb force is neglected for simplicity, and charge-independence of the fundamental forces is assumed. Consider thus the following fundamental nucleon-nucleon interaction:

$$\begin{aligned}\mathscr{V}(ij) = &\{mP_x(ij)+hH(ij)+bP_\sigma(ij)+w\}V_e(ij)\\ &+\{m'P_x(ij)+h'H(ij)+b'P_\sigma(ij)+w'\}V_s(ij)\mathbf{L}_{ij}\cdot(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)\end{aligned} \qquad (1),$$

where m, h, b, w give the proportions of the exchange forces of the Majorana, Heisenberg, Bartlett and ordinary Wigner types, respectively, with

$$m+h+b+w = 1;$$

m', h', b', w' have a similar significance for the character of the spin orbit coupling;

$P_x(ij)$ is the space exchange operator, $P_\sigma(ij)$ is the spin exchange operator, and $H(ij) = P_\sigma(ij)P_x(ij)$ is the operator for the exchange of both spin and space of the particles i, j;

$V_e(ij)$ gives the magnitude and radial dependence of the exchange forces, and shall be assumed to be of Gaussian form

$$V_e(ij) = A_e \exp(-\beta r_{ij}^2);$$

similarly,

$$V_s(ij) = A_s \exp(-\delta r_{ij}^2), \text{ with } r_{ij} = |\mathbf{r}_i-\mathbf{r}_j|.$$

Further $\hbar\mathbf{L}_{ij}$ is the relative orbital angular momentum of particles i, j and may be put more explicitly

$$\hbar\mathbf{L}_{ij} = (\mathbf{r}_i-\mathbf{r}_j)\times(\mathbf{p}_i-\mathbf{p}_j),$$

where $\mathbf{p}_i$ refers to the ith particle momentum.

Finally $\boldsymbol{\sigma}_i$ is the vector spin operator for the ith particle. Eq. (1) is really meant to represent a phenomenological low energy potential.

In terms of the group resonance formalism one considers the following wave function, which is antisymmetrised for the neutrons and for the protons, and assumes the alpha group to be in a spatially symmetric $(1\,s)^4$ configuration:

$$\psi(123, 45) = \{1-H(21)-H(31)\}\chi_\alpha(\widetilde{23}, \widetilde{45})\varphi_\alpha(-1)F(1),$$

where

$$\chi_\alpha(\widetilde{23}, \widetilde{45}) = \tfrac{1}{2}(\alpha(2)\beta(3)-\beta(2)\alpha(3))\,(\alpha(4)\beta(5)-\beta(4)\alpha(5)),$$

with $\alpha(\ )$, $\beta(\ )$ the spin $+1$, -1 wave functions; and $\varphi_\alpha(-1)$ is the symmetric space wave function for the alpha grouping (2, 3, 4, 5), here taken as

$$\varphi_\alpha(-1) = N \exp[-\alpha(r_{23}^2+r_{24}^2+r_{25}^2+r_{34}^2+r_{35}^2+r_{45}^2)],$$

with the normalisation constant $N = 2^{21/4}(\alpha/\pi)^{9/4}$; $F(1)$ is the wave function for the relative motion of the two groups, neutron (1) and centre of gravity of alpha (2, 3, 4, 5). With the present problem it will suffice to use the following representation of $F(1)$:

$$F(1) = \Sigma_{l=0}^{\infty}\Sigma_{j=l-\frac{1}{2}}^{l+\frac{1}{2}} C(j, l)\cdot\frac{f_j(r)}{r}\mathcal{Y}_{j,l,\frac{1}{2}}^{m}(\theta, \varphi),$$

where $\mathbf{r}$, or (r, θ, φ) is the displacement vector of 1 relative to the centre of gravity of group (2, 3, 4, 5); $C(j, l)$ is a coefficient; $f_j(r)/r$ expresses the dependence on r; $\mathcal{Y}_{j,l,\frac{1}{2}}^{m}(\theta, \varphi)$ is the wave function for the vector addition of the neutron spin $\frac{1}{2}$ and the He^5 orbital angular momentum (quantum number l), corresponding to total angular momentum quantum numbers j, m. This leads to the usual form for differential cross-section in terms of partial wave phases by a well known procedure.

Assuming all nucleons to have the same mass M, the total kinetic energy operator K may be written $K = (-\hbar^2/2M)\Sigma_{i=1}^{5}\nabla_i^2$. Thus one assumes the following fundamental Schrödinger equation for the five-body problem:

$$[K-E'+\Sigma_{(12345)}\mathscr{V}(ij)]\,\psi(123, 45) = 0, \qquad (2)$$

where E' is the energy of the system in the laboratory frame, and $\Sigma_{(12345)}$ means the summation over all pairs i, j $(i \neq j)$, from the group (1, 2, 3, 4, 5).

As an additional equation the integral form of the alpha wave equation will be used, with α in $\varphi_\alpha(-1)$ as a variational parameter:

$$\Sigma_{(-1)}^{\sigma}\int d\tau_{-1}\chi_\alpha(\widetilde{23}, \widetilde{45})\varphi_\alpha^*(-1)\{K_\alpha-E_\alpha+\Sigma_{(2345)}\mathscr{V}(ij)\}\chi_\alpha(\widetilde{23}, \widetilde{45})\varphi_\alpha(-1) = 0, \qquad (3)$$

where $\Sigma_{(-1)}^{\sigma}$ refers to a summation over the (2, 3, 4, 5) spin space;

and $\int d\tau_{-1}$ refers to an integration over the alpha (2, 3, 4, 5) configuration space; and K_α is the alpha kinetic energy operator $= (-\hbar^2/2M)\Sigma_{i=2}^{5}\nabla_i^2$; and $(-E_\alpha)$ is the alpha binding energy.

By a procedure which is basically the same as that in the paper of Massey *et al.* [10]), (2) may be simplified to

$$\begin{aligned}&\left\{\frac{\hbar^2}{M}\cdot\frac{5}{8}\left(\frac{d^2}{dr^2}-\frac{l(l+1)}{r^2}\right)+E-E_\alpha\right\}f_j(r)\\&=(4w+2b-m-2h)\left\{\int d\tau_{-1}\,|\varphi_\alpha(-1)|^2V_c(12)\right\}f_j(r)\\&+AV_s'(r)f_j(r)+\int_0^\infty dr'\,k_l(r,r')f_j(r');\end{aligned}\tag{4}$$

the phenomenological He^5 spin orbit coupling potential has been written $V_s'(r)$, which when derived from $\mathscr{V}(ij)$ in eq. (1), is given by

$$V_s'(r)=5\left(w'+b'-\frac{m'+h'}{2}\right)\int d\tau_{-1}\,|\varphi_\alpha(-1)|^2V_s(12)\,\frac{\mathbf{r}\cdot(\mathbf{r}_1-\mathbf{r}_2)}{r^2};$$

E is the 5-body energy in the centre of mass system;

$$A=\begin{cases}0 & \text{for } l=0\\ -(l+1) & \text{for } j=l-\tfrac{1}{2}\\ l & \text{for } j=l+\tfrac{1}{2};\end{cases}$$

moreover,

$$\begin{aligned}&\frac{r_N k_l(r,r')}{\frac{128}{15}\cdot\left(\frac{6\alpha r_N^2}{\pi}\right)^{3/2}}\\&=\left\{B+\frac{608}{225}D(r^2+r'^2)\right\}\times n_l\left(\frac{256\alpha}{75}rr'\right)\exp\left\{-\frac{272\alpha}{75}(r^2+r'^2)\right\}\\&+\left(\frac{8\alpha}{8\alpha-3\beta}\right)C\times n_l\left\{\left(\frac{256\alpha}{75}-\frac{32\beta}{25}\right)rr'\right\}\exp\left\{-\left(\frac{16\beta}{25}+\frac{272\alpha}{75}\right)(r^2+r'^2)\right\}\\&-\frac{49}{48}D\exp\left\{-\frac{272\alpha}{75}(r^2+r'^2)\right\}\alpha\frac{d}{d\alpha}\left\{\frac{n_l\left(\frac{256\alpha}{75}rr'\right)}{\alpha}\right\}\\&-E_e\exp\left\{-\frac{272\alpha}{75}(r^2+r'^2)\right\}\left(\frac{6\alpha}{\gamma+6\alpha}\right)n_l\left\{\frac{128}{225}(\gamma+6\alpha)rr'\right\}\\&\times\left[\exp\left\{-\left(\frac{16\gamma}{225}r'^2+\frac{256\gamma}{225}r^2\right)\right\}+\exp\left\{-\left(\frac{256\gamma}{225}r'^2+\frac{16\gamma}{225}r^2\right)\right\}\right]\end{aligned}\tag{5}$$

where r_N is a radius which will later be identified with the nuclear force range; and

$$B = -\frac{188}{5}\cdot\frac{\alpha\hbar^2}{M} + E - 3(m+w)A_e\left(\frac{4\alpha}{4\alpha+\beta}\right)^{3/2}$$

$$\frac{D}{\alpha} = \frac{64}{5}\cdot\frac{\alpha\hbar^2}{M} \qquad (6)$$

$$C = (4m+2h-2b-w)A_e$$

$$E_e = 3(m+w)A_e\left(\frac{12\alpha}{12\alpha+\beta}\right)^{3/2}$$

$$\gamma = \frac{12\alpha\beta}{12\alpha+\beta}$$

and

$$n_l(b) = i^{l-1}j_l(ib) \text{ with } j_l(\theta) = \sqrt{\frac{\pi\theta}{2}}\,J_{l+1/2}(\theta).$$

Eq. (4) neglects the complex integral interaction term arising from the spin orbit coupling, because it is likely to be a small effect, with A_s small compared with A_e, and with the alpha acting sensibly as a unit particle. This agrees with a finding of Massey *et al.* [11]). The l-dependence of the kernel (5) of the integral interaction operator in (4), is a kind of velocity dependence of the He^5 interaction, which is here distinguished from the usual velocity-dependence, direct dependence on E, by the name „l-dependence".

The ordinary interaction in the wave equation (4) simplifies to

$$\begin{aligned} V(r) &= (4w+2b-m-2h)\int d\tau_{-1}|\varphi_\alpha(-1)|^2 V_e(12) \\ &\quad + 5\Lambda\left(w'+b'-\frac{m'+h'}{2}\right)\int d\tau_{-1}|\varphi_\alpha(-1)|^2 V_s(12)\,\frac{\mathbf{r}\cdot(\mathbf{r}_1-\mathbf{r}_2)}{r^2} \\ &= (4w+2b-m-2h)A_e\left(\frac{32\alpha}{32\alpha+3\beta}\right)^{3/2}\exp\left(-\frac{32\alpha\beta r^2}{32\alpha+3\beta}\right) \qquad (7) \\ &\quad + 5\Lambda\left(w'+b'-\frac{m'+h'}{2}\right)A_s\left(\frac{32\alpha}{32\alpha+3\delta}\right)^{5/2}\exp\left(-\frac{32\alpha\delta r^2}{32\alpha+3\delta}\right). \end{aligned}$$

The variational alpha wave equation (3) simplifies to

$$E_\alpha = \frac{18\alpha\hbar^2}{M} + 6(m+w)A_e\left(\frac{4\alpha}{4\alpha+\beta}\right)^{3/2}.$$

Thus, minimising E_α with respect to α, one obtains

$$\frac{18\alpha\hbar^2}{M} = -9(m+w)A_e\left(\frac{4\alpha}{4\alpha+\beta}\right)^{3/2}\cdot\frac{\beta}{(4\alpha+\beta)}. \qquad (8a)$$

Therefore, more concisely,

$$E_\alpha = \frac{6\alpha\hbar^2}{M} \cdot \left(\frac{\beta - 8\alpha}{\beta}\right), \tag{8b}$$

and

$$B = -\frac{4\left(\alpha + \frac{16}{15}\beta\right)}{(\beta - 8\alpha)} E_\alpha + (E - E_\alpha). \tag{9}$$

3. Relation of Phase Parameters to Interaction Parameters

A new general analysis relating certain useful phase parameters to parameters defining interaction, will now be introduced. This analysis which has several advantages, was used in the larger part of the numerical calculations reported in the present paper.

On rather general lines, one expects a radial wave equation of the following type for the many-nucleon problem, arising from the relative motion of 2 groups of nucleons:

$$\frac{d^2f(r)}{dr^2} + \left(k^2 - \frac{l(l+1)}{r^2}\right) f(r) = U(r)f(r) + \int_0^\infty W_l(r, r')f(r')dr' \tag{10}$$

$$= G(r), \text{ say,}$$

where $W_l(r, r')$ is symmetrical in r, r'; $k^2 = 2M_{\text{red}}E/\hbar^2$, and $U(r) = 2M_{\text{red}}V(r)/\hbar^2$, with E the energy of relative motion of the 2 groups and M_{red} the reduced mass corresponding to the relative motion. For He^5, (10) really corresponds to (4), and then from the previous section, $G(0) = 0$ for the normal condition $f(0) = 0$ and with $W_l(0, r') = 0$. The precise general restriction on fundamental forces which would lead to $G(0) = 0$, is not known, but it is probably true for a general class of bounded short range forces, at least. For short range forces, one similarly considers a nuclear range r_N, such that for $r > r_N$, $G(r)$ is effectively 0. This leaves r_N partly free, but unless otherwise mentioned the smallest possible value shall be taken for r_N.

Now with the boundary conditions: $f(0) = 0$, and $f(r)$ tending to $N_l^{-1} \sin(kr - l\pi/2 + \eta_l)$ as r tends to ∞, one finds a general identity (see, for example, Hulthén [12]):

$$-N_l^{-1}k \cdot \sin \eta_l = \int_0^\infty j_l(kr)G(r)dr = \int_0^{r_N} j_l(kr)G(r)dr \tag{11}$$

where η_l is the phase shift and $j_l(kr)$ is as defined in the previous section. The identity easily follows by substituting for $G(r)$ from (10), by partial integrations and insertion of the boundary conditions.

Consider the following outside solution for $f(r)$ (that is for $r > r_N$):

$$f(r) = N_l^{-1}\{\cos \eta_l j_l(kr) + (-)^l \sin \eta_l j_{-l-1}(kr)\}.$$

This tends to the above mentioned asymptotic limit, as may be readily shown. Now select

$$N_l = \cos \eta_l j_l(kr_N) + (-)^l \sin \eta_l j_{-l-1}(kr_N). \tag{12}$$

Then $f(r_N) = 1$. Thus the boundary conditions for the inside solution ($r < r_N$) for $f(r)$ are independent of k and η_l : $f(0) = 0$ and $f(r_N) = 1$. Thus $f(r)$ will only depend on k^2 through the wave equation. This dependence is naturally diminished by the strength of the interaction and the limitation of the energy range. The phases for He^5 indicate strong interaction. Thus $f(r)$ and hence $G(r)$ may be expected to show a limited variation over the energy range considered. Equation (11) is an integral equation for $G(r)$ with r_N as a parameter. Thus an appropriate approach to the inside solution, is offered by the expansion of $G(r)$ in terms of a complete set of orthogonal functions, pertaining to the interval $r = 0$ to r_N and the given boundary conditions for $G(r)$. In view of the further use, the natural choice for a set of functions is $j_l(k_n r)$, where k_n is defined by $j_l(k_n r_N) = 0$, and n refers to the number of completed loops of $j_l(k_n r)$ from $r = 0$ to r_N; $j_l(k_n r)$ shall be called the nth harmonic, for short. Put $\theta_n = k_n r_N$, $k_n = n_n k_1$ and $\varrho = k/k_1$.

Thus for $r = 0$ to r_N, one expands

$$G(r) = \Sigma_n A_n j_l(k_n r). \tag{13}$$

The A_n are functions of k^2, and by the above remarks the following expansion is suggested for A_n:

$$\frac{A_n}{k_1^2} = C_{n0} + C_{n1}\varrho^2 + C_{n2}\varrho^4 + \cdots \tag{14}$$

The C_{nm} are clearly dimensionless.

Now by (11),

$$-N_l^{-1} k \sin \eta_l = \Sigma_n A_n \int_0^{r_N} j_l(kr) j_l(k_n r)\, dr$$

$$= \Sigma_n \frac{A_n k_n}{k^2 - k_n^2} j_l(kr_N) j_{l-1}(\theta_n).$$

This may be rewritten

$$\frac{\varrho \sin \eta_l}{N_l j_l(\theta_1 \varrho)} = P(E, r_N) = \Sigma_n \frac{k_n^2}{k_n^2 - k^2} B_n, \tag{15}$$

where[4] $B_n = A_n/k_1 k_n \cdot j_{l-1}(\theta_n)$, and $P(E, r_N)$ is a function of the energy E, or of ϱ; a function, which is known over a limited energy range, except for the parameter r_N. Equation (15) establishes a relation between phases and interactions, through $G(r)$; and already one sees how the effect of the higher harmonics is limited, in the definition of B_n.

The wave equation (10), with (13), is now solvable, and subject to the boundary conditions the inside solution reads

$$f(r) = \frac{j_l(kr)}{j_l(kr_N)} + \Sigma_n \frac{A_n}{k^2 - k_n^2} j_l(k_n r). \tag{16}$$

As required, this equation has no explicit reference to η_l. The inside and outside solutions can be shown to join smoothly at $r = r_N$. The present paper shall not undertake to derive the convergence properties of this or any other solution; but it should be mentioned that bounded and relatively smooth $G(r)$ with a limited number of loops are visualised, and that the energy range, at least for the He^5 problem shall make $\varrho < 1$ in general.

Now

$$\Sigma_n A_n j_l(k_n r) = G(r) = U(r) f(r) + \int_0^\infty W_l(r, r') f(r') dr'.$$

This with (16) relate the A_n's to the interaction. In principle this can give the necessary number of linear equations in A_n's to solve for the significant harmonic amplitudes in terms of the interaction parameters, at a given k. This together with (15) relates phases to interaction parameters, as a function of energy. To be at all practicable one would in general require $U(r)$ and $W_l(r, r')$ to be parametrised in a relatively simple way. By inspection of the probable nature of $G(r)$ and the above mentioned limitation on the effect of higher harmonics on phases, one can decide on the number of significant harmonics.

3(a). VELOCITY-INDEPENDENT INTERACTION, GENERAL l

Since $W_l(r, r')$ is not readily parametrised for the He^5 problem, and the Introduction indicates that a good first approximation here may be had with no integral interaction, this case shall be taken. Thus from now on one amends (10) to

$$G(r) = U(r) f(r). \tag{10a}$$

This covers a fairly large class and the interaction is velocity-independent, or $U(r)$ independent of E.

Thus

$$U(r)\left\{\frac{j_l(kr)}{j_l(kr_N)}+\Sigma_n\frac{A_n}{k^2-k_n^2}j_l(k_nr)\right\}=\Sigma_nA_nj_l(k_nr). \tag{17}$$

Now in view of the limited emergy domain considered, and (14), one may proceed to the following power series expansions:

$$\frac{k_n^2}{k_n^2-k^2}=\frac{1}{1-\frac{1}{n_n^2}\varrho^2}=1+\frac{1}{n_n^2}\cdot\varrho^2+\frac{1}{n_n^4}\cdot\varrho^4+\cdots$$

and

$$B_n=\frac{j_{l-1}(\theta_n)}{n_n}\{C_{n0}+C_{n1}\varrho^2+C_{n2}\varrho^4+\cdots\},$$

by (14). Therefore, by (15),

$$P(E,r_N)=\Sigma_n\frac{j_{l-1}(\theta_n)}{n_n}\left\{C_{n0}+\left(C_{n1}+\frac{C_{n0}}{n_n^2}\right)\varrho^2+\cdots\right\}. \tag{18}$$

If now all the functions of k in (17) are similarly expanded as power series in ϱ^2, and the coefficients of like powers extracted, one obtains the following relations, giving the two simpler cases as examples:

$$\frac{U(r)}{k_1^2}\left\{\left(\frac{r}{r_N}\right)^{l+1}-\Sigma_n\frac{C_{n0}}{n_n^2}j_l(k_nr)\right\}=\Sigma_nC_{n0}j_l(k_nr), \tag{19a}$$

$$\begin{aligned}\frac{U(r)}{k_1^2}&\left\{-\left(\frac{r}{r_N}\right)^{l+3}\cdot\frac{\theta_1^2}{2(2l+3)}-\Sigma_n\frac{j_l(k_nr)}{n_n^2}\left(C_{n1}+\frac{C_{n0}}{n_n^2}-\frac{\theta_1^2C_{n0}}{2(2l+3)}\right)\right\}\\&=\Sigma_nj_l(k_nr)\left(C_{n1}-\frac{\theta_1^2C_{n0}}{2(2l+3)}\right).\end{aligned} \tag{19b}$$

Equation (19a) shows the function of the C_{n0} as interaction parameters. If one uses (19a, b) to eliminate $U(r)/k_1^2$ between them, and integrate the resultant equation from 0 to r_N, one obtains the following general relation

$$\Sigma_nj_{l-1}(\theta_n)\left\{\frac{C_{n1}}{n_n}-\frac{C_{n0}}{n_n^3}\right\}=\frac{\theta_1}{2}\Sigma_n\{j_{l-1}(\theta_n)\}^2\frac{C_{n0}^2}{n_n^4}. \tag{20}$$

Equation (20) may be used to simplify the second term of $P(E,r_N)$ in (18):

$$P(E, r_N) = \Sigma_n j_{l-1}(\theta_n) \cdot \frac{C_{n0}}{n_n} + \Sigma_n \left\{ 2[j_{l-1}(\theta_n)] \frac{C_{n0}}{n_n^3} + \frac{\theta_1}{2} [j_{l-1}(\theta_n)]^2 \cdot \frac{C_{n0}^2}{n_n^4} \right\} \varrho^2 + \cdots \tag{21}$$

Thus the first two terms of $P(E, r_N)$ depend only on C_{n0}. The linear equations (19a) relate $U(r)/k_1^2$ to the C_{n0}, and thus define the first two terms in the expansion of $P(E, r_N)$, which is related to phases by (15). Thus at low energies, if $P(E, r_N)$ may be usefully represented by two terms in its energy expansion, this scheme relates phases to interaction, at all l. This analysis may readily be taken to higher values of energy, or higher order terms in the expansion of $P(E, r_N)$, except that the higher the order the more of C_{nm} are required. Thus for $P(E, r_N)$ up to ϱ^{2s} one requires C_{nm} up to the order $m = s-1$ for $s \geqq 1$, if $P(E, r_N)$ is simplified by relations such as (20). Thus one needs s of the equations (19) to relate these C_{nm} to $U(r)/k_1^2$.

As an example, the relevant relations up to the fourth term in $P(E, r_N)$ shall be given for the case $l = 0$. The case $l = 1$ has also been worked out and used, and leads to similar relations. These and those for higher l may be induced from those for $l = 0$, with the help of the general form, for arbitrary l, for the lower order terms given above.

If $l = 0$, one has $j_0(kr) = \sin(kr)$. Thus $\theta_n = n\pi$, and $n_n = n$. As for (19), one arrives at

$$\frac{U(r)}{k_1^2} \left\{ \left(\frac{r}{r_N} \right) - \Sigma_n \frac{C_{n0}}{n^2} \sin(k_n r) \right\} = \Sigma_n C_{n0} \sin(k_n r), \tag{22a}$$

$$\frac{U(r)}{k_1^2} \left\{ - \left(\frac{r}{r_N} \right)^3 \cdot \frac{\pi^2}{3!} - \Sigma_n \frac{\sin(k_n r)}{n^2} \left(C_{n1} + \frac{C_{n0}}{n^2} - \frac{\pi^2}{3!} C_{n0} \right) \right\} = \Sigma_n \left(C_{n1} - \frac{\pi^2}{3!} C_{n0} \right) \sin(k_n r), \tag{22b}$$

$$\frac{U(r)}{k_1^2} \left\{ \left(\frac{r}{r_N} \right)^5 \cdot \frac{\pi^4}{5!} - \Sigma_n \frac{\sin(k_n r)}{n^2} \left[C_{n2} + \frac{C_{n1}}{n^2} - \frac{\pi^2}{3!} C_{n1} + \frac{C_{n0}}{n^4} + \frac{\pi^4}{5!} C_{n0} - \frac{\pi^2}{3!} \cdot \frac{C_{n0}}{n^2} \right] \right\} = \Sigma_n \sin(k_n r) \left(C_{n2} - \frac{\pi^2}{3!} C_{n1} + \frac{\pi^4}{5!} C_{n0} \right). \tag{22c}$$

And for $P_0(E, r_N)$:

$$\frac{\varrho \sin \eta_0}{N_0 \sin (kr_N)} = P_0(E, r_N) =$$

$$\Sigma_n \left[\begin{array}{l} (-)^n \frac{C_{n0}}{n} + \left\{ (-)^n \frac{2C_{n0}}{n^3} + \frac{\pi}{2} \cdot \frac{C_{n0}^2}{n^4} \right\} \varrho^2 \\ + \left\{ (-)^n \frac{2C_{n0}}{n^5} + (-)^n \frac{C_{n1}}{n^3} + \frac{\pi}{2} \left(\frac{C_{n0}^2}{n^6} + \frac{C_{n0} C_{n1}}{n^4} \right) \right\} \varrho^4 \\ + \left\{ \begin{array}{l} (-)^n \frac{2C_{n0}}{n^7} + (-)^n \frac{C_{n1}}{n^5} + (-)^n \frac{C_{n2}}{n^3} \\ + \frac{\pi}{2} \left(\frac{C_{n0}^2}{n^8} + \frac{C_{n0} C_{n1}}{n^6} + \frac{C_{n0} C_{n2}}{n^4} \right) \end{array} \right\} \varrho^6 + \cdots \end{array} \right] \tag{23}$$

and here by (12), $N_0 = \sin (kr_N + \eta_0)$. Equation (23) shows that the higher the order of the term in the ϱ^2 power expansion of $P_0(E, r_N)$, the less do higher n terms contribute relative to the corresponding lower n term (especially $n = 1$). Eq. (23) also shows that contributions of a given C_{nm} are weighed down by at least n times. This and the fact that a reasonable fundamental model for $U(r)$ will only lead to a limited number of loops for $G(r)$, means that one can neglect the contributions after a given n, and also that the accuracy with which C_{nm} need be calculated may be less the higher n. This reduction of the effect of higher harmonics is most effective for $l = 0$, because for higher l, $n_n < n$ (for $n > 1$).

The practical analysis of $P(E, r_N)$ in terms of a limited number of ϱ^2 powers would always involve some approximation, due to the neglect of higher powers, especially in the coefficients obtained for the higher powers retained. To avoid this somewhat and also to relate this to a more useful analysis of phases, the following development is introduced.

3(b). RELATION OF PHASE PARAMETERS TO POTENTIAL PARAMETERS

In general terms for (15), subsection 3(a) traces the ϱ^2 power expansion of the right hand side for the case of a velocity-independent interaction potential. This subsection shall indicate the expansion of the left hand side of (15).

On account of (12), one has

$$P^{-1}(E, r_N) = \frac{N_l j_l(kr_N)}{\varrho \sin \eta_l}$$

$$= \frac{j_l(kr_N)}{\varrho} \{\cot \eta_l j_l(kr_N) + (-)^l j_{-l-1}(kr_N)\} \tag{24}$$

$$= (\varrho^{2l+1} \cot \eta_l) \left\{\frac{j_l(\theta_1 \varrho)}{\varrho^{l+1}}\right\}^2 + (-)^l \left\{\frac{j_l(\theta_1 \varrho)}{\varrho^{l+1}}\right\} \{\varrho^l j_{-l-1}(\theta_1 \varrho)\}.$$

Now Chew and Goldberger [13]) indicated on quite general assumptions (essentially of strong short range interaction) that

$$\varrho^{2l+1} \cot \eta_l = a_0 + a_1 \varrho^2 + a_2 \varrho^4 + \cdots \tag{25}$$

is a convergent expansion. Similarly the power series expansion of the spherical Bessel functions in curly brackets in (24), near $\varrho = 0$, is well-known. The result for $l = 0$ shall be stated:

$$\pi^{-2} P_0^{-1}(E, r_N) = \left(a_0 + \frac{1}{\pi}\right) + \left(a_1 - \frac{\pi^2}{3} a_0 - \frac{2\pi}{3}\right) \varrho^2$$
$$+ \left(a_2 - \frac{\pi^2}{3} a_1 + \frac{2\pi^4}{45} a_0 + \frac{2\pi^3}{15}\right) \varrho^4 \tag{26}$$
$$+ \left(a_3 - \frac{\pi^2}{3} a_2 + \frac{2\pi^4}{45} a_1 - \frac{\pi^6}{315} a_0 - \frac{4\pi^5}{315}\right) \varrho^6 + \cdots$$

$P_0(E, r_N)$ can be calculated from (26) by finding the reciprocal series. This can be done in most cases — in fact from (23) one expects $(a_0 + 1/\pi) \neq 0$, for cases when it is allowed. In practice the latter can be ensured, by not using r_N equal to the Fermi scattering radius. This is also the radius of the impenetrable sphere representation of zero energy S scattering, and the restriction only arises when this radius is positive. Since r_N is really free within limits, one can always take the reciprocal of (26). This and (23) can then be used by equation of coefficients of like powers of ϱ^2 to relate the C_{nm} directly to the a_r. A number of equations are obtained which do not involve energy, and should thus be valid over a larger energy range than some of the expansions may warrant. In practice, with actual numerical values, this reciprocal may be effected rather simply, and thus only the first two equations obtained in the general case will be quoted:

$$\pi^{-2} \left(a_0 + \frac{1}{\pi}\right)^{-1} = \Sigma_n (-)^n \frac{C_{n0}}{n}, \tag{27a}$$

$$-\pi^{-2}\left(a_0+\frac{1}{\pi}\right)^{-2}\left(a_1-\frac{\pi^2}{3}a_0-\frac{2\pi}{3}\right)=\Sigma_n\left\{(-)^n\frac{2C_{n0}}{n^3}+\frac{\pi}{2}\frac{C_{n0}^2}{n^4}\right\}. \quad (27b)$$

Of course other similar relations may also be derived by taking the reciprocal of (23) instead. The case taken merely depends on whether $U(r)$ or η_l is known. Equations such as (27) relate phase parameters to potential parameters. As such it must be joined to (22a), for example, and (25) to connect phases to potentials. This scheme applies to the general class of finite range velocity-independent potentials. It has also been established for $l = 1$,and may quite simply be derived also for higher l.

In the case of He^5, $l = 0$, the analysis was related to the experimental phases by starting from

$$(E'_n)^{1/2}\cot\eta_0 = C_0+C_1E'_n+C_2(E'_n)^2+\cdots, \quad (28a)$$

where E'_n is the neutron energy in the laboratory frame in MeV. In view of the convergence of (28a) and the limited experimental accuracy, one takes only as many terms in (28a) as is necessary to represent $(E'_n)^{1/2}\cot\eta_0$ within the experimental error. This type of analysis is familiar from the shape-independent analysis of nucleon-nucleon scattering data, and is a useful and practical procedure giving some indication as to how much information the phase data can give. The C_r are proportional to the a_r, the proportionality factor depending on a power of r_N. Thus the general analysis (28a) may be made, and a_r deduced for each r_N considered. The corresponding practical phase analysis used here for $l = 1$, analyses $(E'_n)^{3/2}\cot\eta_1$ in terms of a similar E'_n power series

$$(E'_n)^{3/2}\cot\eta_1 = C_0+C_1E'_n+C_2(E'_n)^2+\cdots \quad (28b)$$

One can briefly indicate two types of uses of the above analysis:

(a) If $U(r)$ and hence r_N is known, then, e.g. for $l = 0$, (22) can give a sufficient number of linear equations (by reading this equation at a given set of values of r, say) to determine the significant harmonic amplitudes C_{n0}, C_{n1}, etc. — as may be required. (23) will then give $P_0(E, r_N)$ as a power series, which may be inverted, and by comparison with (26) will yield a number of linear equations for the a_r. Then C_r may be deduced and hence phases versus energy by (28a).

(b) If the phases are known over, say, a limited energy domain above $E'_n = 0$: If one is after a compromise fit with a given type of $U(r)$, (a) may be applied in a step by step procedure, to obtain

a best fit. For sufficiently accurate data a more purely phenomenological procedure may be of interest. The analysis of (28) is performed. Say, one finds three significant phase parameters (C_0, C_1, C_2, as for the $P_{3/2}$ date below) are needed to represent the phases. A possible procedure may then be: If consideration of the likely fundamental interaction model indicates, say, that the interaction potential is likely to have at most 2 loops round the zero potential axis (as for $P_{3/2}$ case below), then in terms of the effect on phases, the 2 lower harmonics may suffice. Of course practically arbitrary values of higher harmonics may be chosen to give the same phases, but unless they are small relative to the 2 lower harmonics they would result in more loops or wiggles in the potential than the fundamental model can cater for. Thus 2 harmonics will give the smoothest and simplest potential with the required number of parameters (2 harmonics and the range r_N). This principle of selection of one of an arbitrary set of possible cases, may be termed the principle of simplicity. Its basis should be general notions on the interaction model, together with the number of available phase parameters. Thus consider the two lower harmonics to be the only ones. Select a suitable r_N and from the phase parameters deduce the a_r and hence by (27) the C_{10}, C_{20}. The latter equation is a quadratic and one set (or even in some cases both) may be eliminated on physical grounds; for example one set may lead to bound states, as below for the $P_{3/2}$ analysis. Thus (22a) yields $U(r)$, and hence (22b) the required C_{n1}. Hence (23) and (26) may be used to deduce a_2 and hence C_2. This may then be compared with the experimental value. By taking a number of r_N one may deduce that r_N for which C_2 is correctly calculated. The $U(r)$ corresponding to the latter case gives the required potential. Since one cannot exclude small amplitude higher harmonics, it may then be necessary and useful to try and eliminate such small, unlikely, peculiarities as the use of only 2 harmonics may give rise to. The final potential may then be tested as in (a).

A useful function is the inside wave function at zero energy

$$f(r) = \left(\frac{r}{r_N}\right)^{l+1} - \Sigma_n \frac{C_{n0}}{n_n^2} j_l(k_n r). \tag{29}$$

Here one notes how the effect of higher harmonics are weighed down. Hence by (19a) one sees how higher periodicity oscillations in $U(r)$ correspond almost directly to higher harmonics.

This general method has been applied in one or other of the forms (a), (b) above, for S waves up to 3 phase parameters and for P waves up to 4. It appears quite practical.

4. The Repulsive Core Interpretation

The first alternative, indicated by the S wave data, considers an ordinary velocity-independent potential with a repulsive core and spin orbit coupling.

Introduce the He^5 interaction potential

$$V(r) = V_1(r)+\langle \boldsymbol{\sigma} \cdot \mathbf{L} \rangle V_2(r), \tag{30}$$

where $\langle \boldsymbol{\sigma} \cdot \mathbf{L} \rangle$ is the expectation value of the spin orbit coupling operator, with $\boldsymbol{\sigma}$ the neutron Pauli spin vector, and $\hbar\mathbf{L}$ the He^5 orbital angular momentum. By the basic idea of the introduction $V_1(r)$ and $V_2(r)$ are supposed to be velocity-independent, and also specifically l-independent. The scattering is analysed in terms of states corresponding to j, m, l, s (for spin) as good quantum numbers.

Hence by the vector model it easily follows

$$V(r) = V_1(r)+\Lambda V_2(r), \tag{31}$$

where Λ has already been defined in Section 2. Thus $V_0 = V_1(r)$ is the S wave potential; and $V_{3/2} = V_1(r)+V_2(r)$ and $V_{1/2}=V_1(r)-2V_2(r)$ the $P_{3/2}$ and $P_{1/2}$ potentials, respectively. In line with the development of the knowledge of nuclear forces, or in fact for simplicity, it is assumed that spin orbit coupling is a secondary effect, such that at least for the domain where $V_1(r)$ must be considerable, $V_2(r)$ is only a small fraction of it. This seems in accord with the phases. On the other hand, however, $V_2(r)$ is known [14]) to be much larger (some 30 times) than indicated by the relativistic Thomas-type model for spin orbit coupling. Thus, since there is no preliminary prescription for the shape of $V_2(r)$, it shall be considered a simple well because the theoretical and experimental accuracy possible at present cannot pretend to insist on a complex interaction shape.

From the phenomenological viewpoint, a $V_1(r)$ with a predominant repulsive core and with an attractive well outside, or with two loops, at the very least, would seem a natural choice to lead to the considerable negative S phases, with considerable positive values for both P phases. The attractive well is demanded by P phases because $V_2(r)$ adds with opposite sign for the two P waves.

To establish a fundamental model for the $V_1(r)$ found, one would be limited to the simplest forms for $V_1(r)$. From a fundamental point of view, a repulsive core of $V_1(r)$ would correspond to the neutron-saturated core of the alpha particle. But 2 pronounced loops for $V_1(r)$ may already be more than one usually bargains for. Reference to the ordinary interaction term in eq. (4) shows that 2 loops in $V_1(r)$ would require either a pronounced double loop, with a very pronounced repulsive core, for $V_e(12)$ (in the face of the averaging due to the motion of the alpha constituents, implicit in (4)), or pronounced differences in the ranges of the different types of exchange forces, neither of which complications one would entertain without good reason. To anticipate, on the order of core, it need be said that the absolute minimum radius for the core of $V_1(r)$ is 2.45×10^{-13} cm., which obtains for a $V_1(r)$ without an attractive outside well and with infinite height. Further, $V_e(12)$ is a low energy phenomenological potential, and even the high energy data only suggests a nucleon core radius of the order of 0.5×10^{-13} cm.

The present section is based on box potentials for $V(r)$. Then the phases may be calculated simply. It is suggested that for any particular box potential, moderate rounding off of corners corresponds chiefly to the higher harmonics of Section 3, which have small effect on the phases. Thus box potentials should give the general trends for all potentials with the same number of loops, and excluding long tail potentials. A more detailed investigation of the repulsive core case was not undertaken because the conclusions with box potentials are negative with such marked emphasis, and in any case the fundamental model for the predominant repulsive core interpretation of $V_1(r)$ involves the above mentioned unattractive features.

The different sources of data agree best for the S and $P_{3/2}$ phases, and in fact much of the present discussion need only refer to these and the positive sign of the $P_{1/2}$ phases. The phase analysis below yields 2 and 3 parameters respectively for the S and $P_{3/2}$ phases. Consider thus the box potential of fig. 2 as basic for the latter two waves. Assume an infinite repulsive core. Then r_1 is common to all l, but V_w and r_2 may have different values for S and $P_{3/2}$. This gives the required number of free parameters for S and $P_{3/2}$. The use of an infinite repulsive core minimises the radii concerned, which tend to be too large compared with other notions on the radius of the

alpha particle. Thus one can select a series of (V_w, r_1, r_2) sets which fit the data for each partial wave. Two values of r_2 were taken,

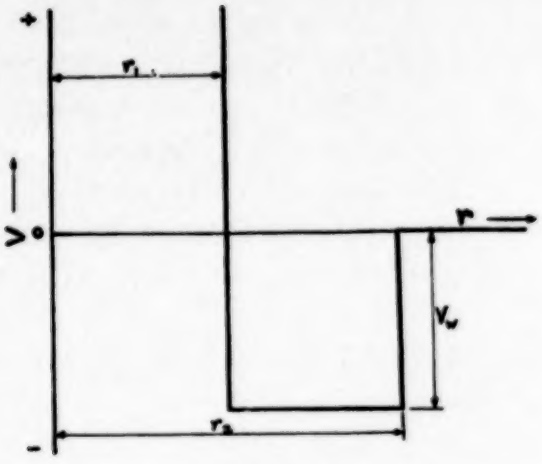

Fig. 2. (n,α) potential with repulsive core.

3.30 and 4.00, with 10^{-13} cm. used in general as the unit of nuclear distances.

Now the following points emerge:

(a) Regarding the goodness of fit of the $P_{3/2}$ phases, with each (V_w, r_1, r_2) set: For a given r_2, the fit is better the smaller r_1, and is best for $r_1 = 0$ or no repulsive core, and for larger r_1 the fit tends to be poor. The $P_{1/2}$ phases indicate a less pronounced, but opposite trend.

A more general approach based on the theory of Section 3, which could show a repulsive core if the data required it, is reported in the next section. It shows for the $P_{3/2}$ wave that the three significant phase parameters lead to a pure well of a type strongly resembling a Gaussian shape, with no core, and $r_N = 5.03$. Only with r_N considerably off this best value (about $r_N = 3.0$) and with C_2 correspondingly off the experimental value, does any tendency to a repulsive core appear.

(b) With a repulsive core, the value of r_1 for S waves cannot be less than 2.45, when $V_w = 0$. The S wave fit is generally better than for the P wave.

(c) With $V_1(r)$ and $V_2(r)$ velocity- and l-independent, and $V_2(r)$ a simple well, one can calculate the minimum value of the $D_{3/2}$ phases by taking the $P_{3/2}$ potential also for the $D_{3/2}$ wave. For both r_2, the smallest $D_{3/2}$ phase obtains for $r_1 = 0$, and is smaller for $r_2 = 3.30$, when it is 9° at $E_n' = 9$ MeV. This is still larger than

suggested by Seagrave [1]), who favoured a value round 0° or even negative. For this at $r_1 = 0$, one would have to consider r_2 smaller than 2.45.

These remarks show the conflicting requirements for the S phases compared with that for the $P_{3/2}$ phase fit and the smallness of the $D_{3/2}$ phases, on the basis of the repulsive core interpretation. Thus on the basis of a velocity-independent interaction this already means that one has to consider the alternative explanation of the S phases. As remarked in the Introduction, it is conceivable that no conflict arises for the predominant repulsive S core case with velocity-dependence.

Even if one neglects the fidelity of the $P_{3/2}$ fit and the $D_{3/2}$ phases, it is found that, with S and $P_{3/2}$ potentials having common (r_1, r_2), a compromise fit of the S, $P_{3/2}$, $P_{1/2}$ phases on the basis of (31) requires, for $V_w(P_{3/2}) = 120$ MeV, that r_2 be a good deal larger than 4.00. For smaller $V_w(P_{3/2})$ the r_2 has to be correspondingly larger, and vice versa. A corresponding conclusion holds for the case of S and $P_{3/2}$ potentials with only r_1 common. Now $r_2 = 4.00$ and $V_w(P_{3/2}) = 120$ MeV are both considerably larger than other notions on the alpha particle would suggest.

On the basis of the above, a repulsive core does not permit of a satisfactory velocity- and l-independent interaction representing the phases. Thus one has to consider the attractive well interpretation of the S data, or $V_1(r)$ negative.

Now by Section 2,

$$V_1(r) = (4w+2b-m-2h)A_e \int d\tau_{-1} |\varphi_\alpha(-1)|^2 v_{12}$$

where $V_e(12) = A_e v_{12}$ in general. Now if one excludes change of sign for the radial dependence v_{12}, and considers v_{12} a simple positive well:

$$V_1(r) < 0 \text{ means } (4w+2b-m-2h)A_e < 0.$$

Therefore the corresponding exchange forces are non-saturating in the absolute sense (Blatt and Weisskopf [15])).

5. General Phenomenological Analysis

5(a). THE PHASE ANALYSIS

The Dodder and Gammel phases, over the energy range $E'_n = 0-10$ MeV, were analysed in terms of the representation

in (28a, b). Due to the divergence of the experimental data, no reliable estimate of the experimental error is available, and the least number of phase parameters had to be decided by judgment. Adequate representation is to be had for all partial waves by 3 parameters, or a parabolic fit, although C_2 for the S wave is so small that even 2 parameters may suffice. The parameters were determined by least square analysis. The means of the magnitudes of deviation from the parabolic fit are 0.9 %, 4.4 %, and 2.6 % respectively for S, $P_{1/2}$ and $P_{3/2}$ wave data. The P wave fit can be improved by giving more weight to the low energy domain.

	C_0	C_1	C_2
S	−2.295	0.1403	0.00271
$P_{1/2}$	11.71	−1.221	0.2635
$P_{3/2}$	3.332	−2.765	0.1702

5(b). THE $P_{3/2}$ PHENOMENOLOGICAL ANALYSIS

The phenomenological procedure sketched as use (b) of the analysis of Section 3 was employed with the above $P_{3/2}$ phase parameters. Thus one determines C_{10}, C_{20}, r_N which will give the experimental C_0, C_1, C_2. Note that 2 harmonics can give a repulsive core potential, if the data required it.

The $P_{3/2}$ case was taken as an example because the divergence between the 2 sources is somewhat less for it as compared with the $P_{1/2}$ wave, and also the $P_{3/2}$ case does not permit of two distinct alternatives as the S wave does. The experimental parameters thus determine:

$$r_N = 5.03, \qquad C_{10} = -0.735, \qquad C_{20} = -0.588.$$

The extra phase parameter over and above those for the shape-independent approximation shows its effect in further specification of the well-shape by the determination of 2 harmonics. The $V(r)$ corresponding to these potential parameters can be deduced from an equation, for $l = 1$, analogous to (22a), and in the main it resembles an attractive Gaussian well. It deviates somewhat from the latter in the direction of the square well, in that significant deviations of $V(r)$ from $V(0)$ start at a larger radius and then $V(r)$ declines more rapidly than the Gaussian, $(V(2.73) = \frac{1}{2}V(0))$. $V(r)$ also has a slight repulsive lip from $r = 4$ to r_N, which is less

than 0.1 $V(0)$ at the maximum. This lip is apparently not an essential feature of the $P_{3/2}$ phenomenological interaction, but rather arises from the restriction to only 2 harmonics in the analysis. Cutting the lip off will probably correspond chiefly to the introduction of higher harmonics with a small effect on the phases. In the above analysis one starts with r_N and the experimental C_0, C_1, and hence calculates $V(r)$ and C_2. Only when r_N is considerably off the above value, round $r_N = 3.0$, does the $V(r)$ tend to show a repulsive core. This would correspond to a poor value of C_2. In fact, already for $r_N = 4.0$, the coefficient of ϱ^4 in $P_1(E, r_N)$ determined from the potential is nearly two times that from the phases.

6. The Attractive Well Analysis

6(a). ATTRACTIVE SQUARE WELL FIT

In view of the indications of the $P_{3/2}$ analysis in Section 5(b), it was decided to see to what approximation the simplest case of attractive wells, namely square wells, covered by (31), with $V_1(r)$ and $V_2(r)$ of the same range, can represent the S, $P_{1/2}$, $P_{3/2}$, phases and the smallness of the D phases. Being a specific well shape, the fit is a compromise one. The phases can be calculated simply from expressions in closed form. In terms of the second alternative explanation of the S phases, one considers that attractive square well which most nearly represents the S phases and has only 1 bound state.

The best fit is judged to be for square wells with

$$V_1 = -39{\cdot}0 \text{ MeV}, \qquad V_2 = -3{\cdot}51 \text{ MeV}$$

from 0—2·32, and = 0 for larger radii. The D phases at $E'_n = 9$ MeV follow from $V(r)$ by (31):

$$\eta(D_{5/2}) = +1{\cdot}7°, \qquad \eta(D_{3/2}) = +0{\cdot}8°.$$

This seems small enough.

The means of the magnitudes of deviation of the calculated phases from the experimental phases at $E'_n = 0, 1, 3, 5, 9$, MeV are as follows: for S wave, 1·0°; for $P_{3/2}$, 10·4°; and for $P_{1/2}$, 8·1°. The P wave fit is distinctly poorer, which is partly due to the chosen well shape, and partly due to the compromise of the requirements of the 3 main partial waves. Even so, the above deviations are within the order of the divergences between Seagrave [1]), and Huber and Baldinger [2])

phases; and this case shows the promise of the attractive well interpretation of the S wave data. The range of 2·32 is perhaps not unreasonable on general notions of the alpha particle radius, and is just less than the minimum radius (2·45) for the repulsive core interpretation of S wave data.

6(b). THE GAUSSIAN FIT

In view of the divergence of the two main sources of phases, it was suggested in the Introduction that it is advisable at present to estabiish some connection with a fundamental theory for the He^5 interaction. Thus, in view of the formulation of Section 2 for the latter, and also in view of the indication of subsection 5(b) for the $P_{3/2}$ phases, it was decided to establish the compromise fit of the S, $P_{1/2}$, $P_{3/2}$, data with attractive Gaussian wells. To use the theory of Section 3, the Gaussians were cut off at $r_N = 2a$, where $y = \exp[-(r/a)^2]$ is suitably small at 0·018. For the procedure of Section 3, the precise behaviour of the well close to $r = r_N$ will depend on the number of harmonics taken. For simplicity and in view of divergences between the experimental sources and the preliminary nature of the theory for the spin orbit coupling, $V(r)$ shall be considered as in (31) with $V_1(r)$ and $V_2(r)$ of the same range.

Essentially then the technique employed is a systematic application of use (a) of the analysis of Section 3, using a number of r_N and $V(0)$ to decide on the best compromise. Four harmonics were used for the S wave and 3 for the P waves, because it gave adequate accuracy for the present. The larger number of harmonics for the S wave is for the adequate representation of the effect of the change of sign of the S wave function in the nuclear domain. The linear equations relating the C_{nm} to $U(r)/k_1^2$ were here obtained by reading equations such as (22) off at definite values of r/r_N. This facilitates the procedure, as the spherical Bessel functions are required only at a small number of values of the argument, and a useful amount of calculation can be done independently of r_N. Note that (22) have useful limiting counterparts at $r = 0$. Thus for S, r/r_N was taken = 0, 0.3, 0.6, 0.8; and for P waves $r/r_N = 0$, 0.3, 0.6. Naturally (22) is satisfied at $r/r_N = 1$.

Then, for a given partial wave and a particular r_N, one can select that $V(0)$ which gives the best compromise fit of the phase data. For convenience and simplicity the following criterion was

used here for this selection: Consider the sum of the first two terms of $P(E, r_N)$ as calculated (a) from the potential, (b) from the phases. Take $V(0)$ for which δ, the magnitude of the deviation of sum (a) and (b), is the same at $E'_n = 0$ and 9 MeV. Future work may well use other criteria specifically adapted to the nature of the data of the particular partial wave. Doing this, the following interesting feature appears: δ increases linearly with r_N for $P_{3/2}$ over the range $r_N = 4.0$ to 4.6; and decreases linearly with r_N increasing over the same range for $P_{1/2}$. A best compromise r_N for the P waves would then be where δ is the same for the two P waves, at $r_N = 4.18$. On the other hand, at a given r_N, by (31) one can calculate that $V_0(0)$, for the S wave, which corresponds to the best compromise $V_{3/2}(0)$, $V_{1/2}(0)$, on the basis of velocity-independence. This $V_0(0)$ is equal to the best compromise $V_0(0)$ by the above procedure, at $r_N = 4.37$. The latter value is near enough to the above value, in view of all the possible inaccuracies at present, to indicate that the mean of the two is probably a good compromise value for the 3 partials, say $r_N = 4.3$. The closeness of the 2 values of r_N is some confirmation of the 2 basic ideas taken together: the attractive well interpretation of the S wave date, and the velocity- and l-independence of the interaction.

Note $a = 2.15$, which is quite near to $a = 2.30$, found by Sack [14]) *et al.* for a Gaussian fit of the P waves alone of the (p,α) scattering data.

The above procedure selected the best $V(0)$ at $r_N = 4.30$ for the two P waves, and then calculated $V_0(0)$ from (31) on velocity-independence, and the S phases from it.

$l = 0$, *S wave*

$$V_0(0) = -50.89 \text{ MeV}.$$

$C_{10} = 0.937$, $C_{20} = 1.096$, $C_{30} = 0.450$, $C_{40} = 0.117$. By (29) it is seen that the zero energy $f_{1/2}(r)$ passes through zero at $r = 2.74$. At low energy the well will act almost as if it had this as a hard sphere radius, with an attractive well outside corresponding to the continuation of the Gaussian. The third order term, C_2, is about a 7 % correction to $(E'_n)^{1/2} \cot \eta_0$ at 9 MeV, or changes the calculated phases by $+1.4°$. Thus, over the present energy range, one need only use 2 parameters from the S wave data. Comparing the calculated (with 2 parameters) with experimental phase values at $E'_n = 0, 1, 2, 3, 6, 9$ MeV, the average deviation magnitude is 2°.

$l = 1$, $P_{3/2}$ *wave*

$$V_{3/2}(0) = -54.96 \text{ MeV}.$$

$C_{10} = -0.559$, $C_{20} = -0.199$, $C_{30} = -0.155$. At 9 MeV, the fourth order term, C_3, causes an 8 % correction to $(E_n')^{3/2} \cot \eta_1$, or $+1.3°$ to phase. Thus one can use 3 phase parameters over the present energy range. Comparing the calculated (with 3 parameters) and experimental phase values, at $E_n' = 0$, 1, 2, 3, 6, 9 MeV, the average deviation magnitude is 3.4°. This is much better than for the square well fit.

$l = 1$. $P_{1/2}$ *wave*

$$V_{1/2}(0) = -42.75 \text{ MeV}.$$

$C_{10} = -0.270$, $C_{20} = -0.0975$, $C_{30} = -0.0504$. At 9 MeV, the fourth order term causes less than 1 % correction to $(E_n')^{3/2} \cot \eta_1$, or a $-0.1°$ phase change. One need take only 3 parameters for the $P_{1/2}$ phase analysis over the present energy range. Thus the mean deviation magnitude of calculated (with 3 parameters) and experimental phase values at $E_n' = 0$, 1, 2, 3, 6, 9 MeV is 7.0°. This is better than for the square well, but is not good mainly on account of the well shape being not the best for the $P_{1/2}$ wave data, and to a lesser degree to the inevitable r_N compromise.

The above findings on the number of significant phase parameters refer to the Gaussian well.

Thus for Gaussian wells of range $r_N = 4.3$ and $a = 2.15$, one has by (31):

$$\begin{aligned} V_1(0) &= -50.89 \text{ MeV}, \\ V_2(0) &= -4.07 \text{ MeV}, \end{aligned} \qquad (32)$$

i.e.

$$V(r) = -(50.89 + \Lambda 4.07) \exp\{-(r/2.15)^2\} \text{ MeV}.$$

$V_1(0)$ agrees reasonably well with the 47.32 MeV of Sack [14]) *et al.* for (p,α) scattering, P waves only, but the $V_2(0)$ as a smaller, difference effect shows a relatively larger divergence ($V_2(0) = -5.85$ MeV for Sack [14])).

6(c). THE FUNDAMENTAL FORCE PARAMETERS

As explained in the Introduction, the integral interaction term is neglected first of all. Thus $V(r)$ of (32) may be correlated with fundamental forces, by using (7), with $E_\alpha = -28.2$ MeV, by (8b), as a subsidiary condition. In the present case, $\delta = \beta$. By the range of $V(r)$ from (7), (32), and the alpha binding,

$$\alpha = 0.0839 \times 10^{26} \text{ cm}^{-2}, \tag{33}$$

$$\beta = \delta = 0.2853 \times 10^{26} \text{ cm}^{-2}. \tag{34}$$

Therefore $\beta^{-\frac{1}{2}} = 1.87 \times 10^{-13}$ cm.

Now, following Buckingham and Massey [16]), we take for $(m+w) - (h+b)$ the ratio of singlet to triplet potentials in even nucleon-nucleon states, 0.6. Therefore,

$$m+w = 0.8, \quad h+b = 0.2. \tag{35}$$

By (33), (34), (35), and (8a),

$$A_e = -47.6 \text{ MeV},$$

i.e.

$$V_e(ij) = -47.6 \exp\{-(r_{ij}/1{\cdot}87)^2\} \text{ MeV}.$$

This is quite close to that indicated by low energy two-body data. Note that Massey *et al.* [10, 11]) assumed $V_e = -45 \exp\{-(r/1.94)^2\}$ MeV.

Thus by (7) and from the value of $V_1(0)$ it readily follows that

$$4w+2b-m-2h = 1.62. \tag{36}$$

For the Serber force this = 1.2, and the forces are thus of a more non-saturating type than the Serber force (see the end of Section 4).

The exchange coefficients are not completely specified, but may be given in terms of, say, w:

$$\begin{aligned} m &= 0.8-w, \\ b &= 0.705-1.25w, \\ h &= -0.505+1.25w. \end{aligned} \tag{37}$$

A mixture with a predominant Wigner force is thus possible. If one takes $w = 0.377$ which corresponds to the value given by Massey *et al.* [11]), then by (37):

$$m = 0{\cdot}423, \qquad h = -0{\cdot}034, \qquad b = 0{\cdot}234$$

This should be compared with the values of Massey *et al.* [11]):

$$m = 0{\cdot}453, \qquad h = 0{\cdot}063, \qquad b = 0{\cdot}107.$$

The latter means a value for the combination (36) = 1.14. It is not clear to what precisely the discrepancies are due. They may be due to the natural possibility, mentioned in the Introduction, of some freedom on the relative contributions of ordinary and integral interactions terms. This freedom is eliminated in the present discussion by the

basic idea of the alpha considered as acting effectively as a single particle.

By (7) and from the value of $V_2(0)$,

$$\left(w'+b'-\frac{m'+h'}{2}\right)A_s = -1.63 \text{ MeV}. \tag{38a}$$

For $w' = 1$, and $m' = h' = b' = 0$,

$$A_s = -1.63 \text{ MeV}. \tag{38b}$$

The corresponding value of Massey *et al.* [11]) is -2.25 MeV.

6(d). THE INTEGRAL INTERACTION TERM

Using the calculated fundamental constants, the integral interaction term indicated by the theory of Section 2, and neglected so far, may be estimated by (4), (5), (6), (9). With the main ordinary interaction as strong as found above, the wave function will show only a relatively small variation in the nuclear domain over a small energy range, say $E'_n = 0$ to 10 MeV. In the definition of $k_l(r, r')$, B is the only term which depends on energy, and it varies by 7 % only over the above energy range. Thus the estimate of the relative importance of this integral term at zero energy should be significant. For this one uses the zero energy wave function in (29). Since the kernel $k_l(r, r')$ has a limited range in r and r', one can estimate the integral term sufficiently well by graphical extrapolation of (29) for $r' > r_N$; and further, the integral term shall only be estimated up to $r = \frac{1}{2}r_N$. In view of this, and to show the relative size,

$$V_I(r) = \frac{\int_0^\infty k_l(r, r')f_{s,l}(r')dr'}{f_{s,l}(r)}$$

shall be compared with $V(r)$, eq. (32), at $r/r_N = 0$, 0.2 and 0.5. This deals with what is considered as the most significant domain for this comparison. At $r = 0$, the limiting value may be calculated mathematically, and it appears that $V_I(r)$ must have a zero slope at $r = 0$; but at other r values the integration was performed graphically.

The factors limiting the magnitude of V_I are:

(i) In $k_l(r, r')$ all terms have factors, n_l, vanishing with r' as $(r')^{l+1}$ (and so does $f_{s,l}(r')$), and another, Gaussian, factor which decays rapidly at larger radius.

(ii) Cancellation between large kernel terms with different signs.
(iii) The change of sign for the S wave function in the nuclear domain means opposite contributions from the resultant 2 domains.

The effect (ii) is the most important one in general, and then (iii) is conclusive for the S wave. The effect (ii) will restrict the domain over which the energy dependence of B may be neglected.

	S			$P_{3/2}$			$P_{1/2}$		
r/r_N	0	0.2	0.5	0	0.2	0.5	0	0.2	0.5
$V_I(r)$	−0.20	−0.97	+3.19	−20.2	−14.6	−3.25	−26.8	−18.8	−4.28
$V(r)$	−50.9	−43.4	−18.7	−55.0	−46.8	−20.2	−42.8	−36.4	−15.7

Thus the kernel term is clearly negligible for S waves. If the ordinary S wave interaction can be determined accurately from the phases, one has all the data to determine the above information on the exchange forces.

For P waves, the integral term is a larger effect. Now $V(0)$ is 2.7 and 1.6 times $V_I(0)$ for $P_{3/2}$ and $P_{1/2}$ respectively; and $V_I(r)$ has a range, b say, just over $\frac{1}{2}$ that of $V(r)$. A parameter such as $V_I(0)b^2$, together with the greater effect of the centrifugal force on the shorter range V_I, indicates that the phase effect of the integral term is relatively small. For the P waves, the effect is secondary, but not negligible. In terms of the analysis of Section 3, the proper detailed evaluation of the effect of the integral term is in terms of its contribution to $G(r)$, etc., but then its dependence on k is required. The present simple comparison, however, already shows the relative size of the term at various radii, and also its limited range, and serves to confirm the basic idea concerning the importance of the integral term. Having determined the fundamental force parameters as above, the best procedure is probably to try it out in the detailed He^5 wave equation, by a numerical solution of same, as by Massey *et al.* [10,11]). This need only be done at 3 energies for the P waves, in view of the fact that 3 phase parameters will define the phase function $(E_n')^{3/2} \cot \eta_1$.

7. Conclusion

The calculations were based on the idea that the alpha particle acts in effect as a unit particle in the scattering, and that hence the He^5 interaction is velocity-independent. This, together with the

idea of an attractive S wave interaction with one bound state, which is unobservable due to its wave function vanishing, leads to a solution which is consistent with the basic idea to a good approximation. The attractive well S wave interaction is here forced on one, since the other alternative of a predominantly repulsive core interaction does not permit of a satisfactory interaction to represent the data. The fundamental exchange force on this picture turns out to be strongly non-saturating in the absolute sense. As is well-known, the calculations [15]) in connection with saturation involve a probably unwarranted extrapolation of the dependence on velocity and nuclear density usually assumed for the forces in ordinary nuclei. In any case the criterion for absolute saturation is too stringent [15]). As it is difficult to arrive at a reliable and useful criterion for conditional saturation, it requires further investigation to decide to what extent the above forces really fail to saturate, and hence calls for ideas such as a short range repulsive core, or non-linearity with respect to nuclear density. A small radius repulsive core, for the He^5 interaction, which may result from the averaging, over the alpha coordinates, of a fundamental nucleon-nucleon force with a core of a radius $= \frac{1}{2} \times 10^{-13}$ cm., may perhaps be incorporated without undue change in, say, the above Gaussian He^5 inter-action, at least for low energy scattering.

The sign of the spin orbit coupling accords with the requirement of the shell model. The magnitude is somewhat larger than the figure usually quoted for the latter. This may be partly due to differences in well shapes, and partly to the involved connection of the present figure for the nucleon-nucleon spin orbit coupling, and that from heavier nuclei on the shell model.

The present work cannot decide the exchange nature of the spin orbit coupling. A symmetric exchange nature, such as the M_2^{ij} of Abraham *et al.* [7]), gives a zero spin orbit coupling for He^5, and is thus not on its own in accord with the data for the latter. Hence there is no need to fear the large effect, which these authors pointed out for M_2^{ij}, on the D binding energy. For the neutral interaction, the present A_s leads to a contribution to the binding energies of D, T and He^4 of only a few percent.

The new method, presented above, for correlating phase parameters (or phases), and potential parameters (or potentials), has proved very useful and simple to apply to the required accuracy.

The author would like to acknowledge the benefit he derived from the advice and encouragement of Professor L. Rosenfeld throughout the present research; and the aid of bursaries of the South African Council for Scientific and Industrial Research.

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Nuclear Physics, North-Holland Publishing Co., Amsterdam, **1** (1956)

THE TWO-BODY SPIN-ORBIT FORCE

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Received 26 January 1956

Abstract: A first order estimate is made of the two-body spin-orbit force from the ^{2}P splitting of Li^7. Considered together with recent work on He^5, the estimated spin-orbit force makes a large change in the calculation of the binding energy of the deuteron.

1. Introduction

In an earlier study [1]) of nuclear interactions a calculation was made of the binding energies of H^3 and He^4 with a two-body potential made up of central and tensor forces and consistent with two-body data. It was suggested there that for accurate calculations it would be necessary to consider also a spin-orbit force of the type

$$(V_m+V'_m P^\tau_{ij})(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)\cdot(\mathbf{r}_i-\mathbf{r}_j)\times(\mathbf{p}_i-\mathbf{p}_j)\hbar^{-1}J_{ij} \qquad (1)$$

where $2P^\tau_{ij}=1+(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j)$ and J_{ij} gives the radial dependence. Expression (1) and the tensor force are the only types of non-central forces allowed by charge-independence and general invariance requirements [2]). Both these forces occur also in potentials derived from field theory. In the latter theory the radial dependence J_{ij} is complicated and uncertain at short distances and it seems preferable at this stage to assume a simple well shape such as the Gaussian or Yukawa potential.

In this paper a first order calculation is made of the splitting by the force (1) of the lowest ^{2}P state of Li^7. In the light nuclei this is the first stable level which is split by non-central forces.

2. The Wave-Functions

It is assumed that the central part of the nuclear interaction predominates and the spin-orbit and tensor couplings are perturbations. In a first order estimate of the spin-orbit force the tensor force can be neglected. Moreover in place of the exact eigenfunction of the central force, the Hartree approximation with the configuration $(1s)^4(2p)^3$ is used as suggested by the shell model. The

wave-function for the $(1s)^4$ configuration can be written as [1])

$$\psi_S^{(4)} = \{(\sigma_1 \cdot \sigma_3) - (\tau_1 \cdot \tau_3)\}\varphi^{(4)}\chi^{(4)} \exp\{-R^{-2}(r_1^2+r_2^2+r_3^2+r_4^2)\} \quad (2)$$

where

$$\begin{aligned} \varphi^{(4)} &= \tfrac{1}{2}(\alpha_1\beta_2-\alpha_2\beta_1)(\alpha_3\beta_4-\alpha_4\beta_3), \\ \chi^{(4)} &= \tfrac{1}{2}(p_1n_2-p_2n_1)(p_3n_4-p_4n_3), \end{aligned} \quad (3)$$

α, β referring to the two states of spin $\pm\frac{1}{2}$ and p, n to the proton, neutron states of a nucleon. The $(2p)^3$ wave-function is

$$\begin{aligned} \psi_P^{(3)} &= [P_5, P_6, P_7]\{(\sigma_5 \cdot \sigma_7) - (\tau_5 \cdot \tau_7)\}(\alpha_5\beta_6-\alpha_6\beta_5)\alpha_7(p_5n_6-p_6n_5)n_7 \\ &\quad \times \exp\{-R^{-2}(r_5^2+r_6^2+r_7^2)\} \end{aligned} \quad (4)$$

where

$$\begin{aligned} [P_5, P_6, P_7] &= X_5(2X_6Y_7-Z_6Z_7)+X_6(2X_7Y_5-Z_7Z_5) \\ &\quad +X_7(2X_5Y_6-Z_5Z_6), \\ X = -\frac{1}{\sqrt{2}}(x+iy), &\qquad Y = \frac{1}{\sqrt{2}}(x-iy), \qquad Z = z. \end{aligned} \quad (5)$$

The space part of (4) is symmetrical in the 3 particles [3]). Eq. (4) represents a $P_{3/2}$ state with $T = \frac{1}{2}$, $T_z = -\frac{1}{2}$. The totally antisymmetrical wave-function for the seven particle system is then

$$\psi^{(1)} = cP_7P_6P_5\psi_P^{(3)}\psi_S^{(4)} \quad (6)$$

where c is the normalisation constant and

$$P_n = 1-\sum_{i=1}^{n-1} P_{in} \quad (7)$$

which transforms a wave-function antisymmetrical in $(n-1)$ particles into one antisymmetrical in all n [4]). Owing to the very large number of terms in the expanded form of (6) it is convenient to arrange the terms in a series of orthogonal terms, by choosing suitable orthogonal representations of the permutation group [5,6]). The method used in this paper is illustrated in the simple case of the configuration $(1s)^4(2p)^1$. By means of the invariant spin operators

$$\begin{aligned} \sigma_{ij} &= \sigma_i \cdot \sigma_j, \\ \sigma_{ijk} &= i\sigma_i \cdot (\sigma_j \times \sigma_k), \end{aligned} \quad (8)$$

the vector space of spin functions with $S = \frac{1}{2}$, $S_z = \frac{1}{2}$ is spanned by the five orthogonal vectors obtained by operating on $\varphi^{(4)}\alpha_5$ with 1, σ_{13}, σ_{15}, σ_{35} and σ_{135}. The $P_{3/2}$ state with $T = \frac{1}{2}$, $T_z = -\frac{1}{2}$ can be written as

$$\begin{aligned}\psi^{(5)} = &[(X_1-X_2)\{\sigma_{15}(\tau_{13}-\tau_{135})-\tau_{15}(\sigma_{13}-\sigma_{135})+\tau_{35}-\sigma_{35}\}\\ &+(X_3-X_4)\{\sigma_{35}(\tau_{13}+\tau_{135})-\tau_{35}(\sigma_{13}+\sigma_{135})+\tau_{15}-\sigma_{15}\}\\ &+(X_1+X_2-X_3-X_4)\{\sigma_{15}\tau_{35}-\tau_{15}\sigma_{35}+\sigma_{135}-\tau_{135}\}\\ &+(X_1+X_2+X_3+X_4-4X_5)\{\tau_{13}-\sigma_{13}\}]\\ &\times\varphi^{(4)}\chi^{(4)}\alpha_5\eta_5\ \exp\{-R^{-2}\textstyle\sum_{i=1}^{5}r_i^2\}.\end{aligned} \tag{9}$$

It is easily seen that the four terms in (9) are orthogonal. Moreover the square modulus of each of these terms is the same. Equation (9) cannot be used to represent the He^5 ^2P state since it is a virtual level. The equation (6) representing the $^2P_{3/2}$ state of Li^7 can be expanded in the same way as (9).

3. The Spin-Orbit Energy

The first order estimate of the spin-orbit perturbation of the $^2P_{3/2}$ level is just the expectation value of (1), for the wave-function (6). For simplicity in integration, J_{12} is taken to be $\exp(-R_m^{-2}r_{12}^2)$. From the Landé interval rule it is known that the $^2P_{3/2}-{}^2P_{1/2}$ splitting is three times the $^2P_{3/2}$ perturbation. This splitting is then given by

$$V_m(6M_{sp}+2M_{pp})+V_m'(6M_{sp}-2M_{pp}) \tag{10}$$

where

$$\begin{aligned}M_{sp} &= \langle X_1 e^{-R^{-2}(r_1^2+r_2^2)}|L_{12,z}|X_1 e^{-R^{-2}(r_1^2+r_2^2)}\rangle\\ &= (1+x)^{-5/2},\\ M_{pp} &= \langle X_1X_2 e^{-R^{-2}(r_1^2+r_2^2)}|L_{12,z}|X_1X_2 e^{-R^{-2}(r_1^2+r_2^2)}\rangle\\ &= 2(1+x)^{-7/2},\end{aligned} \tag{11}$$

$L_{12,z}$ being the z-component of $\hbar^{-1}(\mathbf{r}_1-\mathbf{r}_2)\times(\mathbf{p}_1-\mathbf{p}_2)$ and $x=(R/R_m)^2$. The first part of (10) corresponds to the interaction between s and p particles and the second between p particles. In order to assign a value to the parameter x in (11) it is necessary to consider the Coulomb and binding energies.

4. The Coulomb Energy

The Coulomb energy operator is

$$E_{ij}^c = \tfrac{1}{4}(1+\tau_{iz})(1+\tau_{jz})e^2/r_{ij} \tag{12}$$

Using (5) for the Li^7 ground state and the same equation with a change only of T_z from $-\frac{1}{2}$ to $+\frac{1}{2}$ for the Be^7 ground state the Coulomb energies for these mirror nuclei are given by

$$4C_{sp}-2C_{sp}^{ex}+C_{ss}+C_{pp} \tag{13}$$

where

$$\begin{aligned} C_{sp} &= \langle Z_1 e^{-R^{-2}(r_1^2+r_2^2)} | e^2/r_{12} | Z_1 e^{-R^{-2}(r_1^2+r_2^2)} \rangle = \tfrac{5}{3}\pi^{-1/2} e^2/R, \\ C_{sp}^{ex} &= \langle Z_1 e^{-R^{-2}(r_1^2+r_2^2)} | e^2/r_{12} | Z_2 e^{-R^{-2}(r_1^2+r_2^2)} \rangle = \tfrac{1}{3}\pi^{-1/2} e^2/R, \\ C_{ss} &= \langle e^{-R^{-2}(r_1^2+r_2^2)} | e^2/r_{12} | e^{-R^{-2}(r_1^2+r_2^2)} \rangle = 2\pi^{-1/2} e^2/R, \\ C_{pp} &= \langle [P_1, P_2, P_3] | e^2/r_{12} | [P_1, P_2, P_3] \rangle = 1.7\pi^{-1/2} e^2/R. \end{aligned} \tag{14}$$

From the experimental value 1.65 MeV for the Coulomb energy difference Be^7-Li^7, the value of R is found to be 2.3×10^{-13} cm.

An estimate of R is also found from a binding energy calculation. The central potential used is the symmetric potential [7])

$$-\tfrac{1}{3}V_c(\sigma_i\cdot\sigma_j)(\tau_i\cdot\tau_j)\{1-\tfrac{1}{2}g+\tfrac{1}{2}g(\sigma_i\cdot\sigma_j)\}\exp(-r_{12}^2R_c^{-2}), \tag{15}$$

$$V_c=-45\text{ MeV}, \qquad g=0.2, \qquad R_c=1.91\times10^{-13}\text{ cm},$$

which fits two-body scattering data. The maximum binding energy of 12 MeV corresponds to $R=2.5\times10^{-13}$ cm. This agrees with the results of Carroll [8]) allowing for the difference in the interaction parameters. The very low binding energy reveals the approximate nature of the wave-function used here. Carroll has shown that a mixture of several configurations raises the binding energy to about 24 MeV.

5. Estimate of Spin-Orbit Coupling Strength

The $^2P_{3/2}-{}^2P_{1/2}$ energy difference is found from (10) and (11) to be

$$0.76\,V_m+0.44\,V_m' \tag{16a}$$

$$\text{or} \qquad 0.55\,V_m+0.34\,V_m' \tag{16b}$$

according as $R=2.3$ or 2.5×10^{-13} cm, assuming R_m to be 1.87×10^{-13} cm [9]). The values (16) show that the splitting is quite sensitive to the variation of the wave-function parameter R. With the approximations adopted here, namely the shell-model wave-function and neglect of tensor forces expression (16) represents the observed value of -0.477 MeV. From a study of He^5 data, using the same spin-orbit force as used here with the same value of R_m, van der Spuy [9]) finds the relation †

$$V_m+0.5\,V_m'=-1.63\text{ MeV}. \tag{17}$$

From (17) and equating (16) to -0.477 MeV,

† This result corrects a remark made in an earlier paper [1]) that the neutral spin-orbit force, that is expression (1) with $V'_m=0$, has the same effect in He^5 as the exchange spin-orbit force, that is expression (1) with $V_m=0$.

$$V_m = -8 \text{ MeV}, \qquad V'_m = 12.7 \text{ MeV} \tag{18a}$$

$$\text{or} \qquad V_m = -4.8 \text{ MeV}, \qquad V'_m = 6.3 \text{ MeV} \tag{18b}$$

according as $R = 2.3$ or 2.5×10^{-11} cm.

The change in the deuteron binding energy due to the force (1) has been calculated earlier [1]) treating the spin-orbit force as a perturbation on the central and tensor forces. It is approximately

$$0.08\,(V_m - V'_m). \tag{19}$$

Using (18) this is 1.7 or 0.9 MeV for $R = 2.3$ or 2.5×10^{-13} cm. This reinforces the suggestion made earlier [1]) that a calculation of the binding energy of the deuteron must take into account the spin-orbit force. However it must be emphasized here that a higher order calculation which includes the effect of tensor forces may drastically alter the first order estimates of V_m and V'_m.

The author gratefully acknowledges the guidance of Prof. L. Rosenfeld and helpful discussions with Dr. K. J. Le Couteur.

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ON THE RATIO BETWEEN THE ACTIVATION CROSS SECTIONS OF ^{84}Sr AND ^{88}Sr FOR SLOW NEUTRONS

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Received 6 February 1956

Abstract: Samples of $SrCO_3$, irradiated in a nuclear reactor, have been used to determine the ratio between the activation cross sections of the two processes $^{84}Sr(n, \gamma)^{85}Sr$ (65 d) and $^{88}Sr(n, \gamma)^{89}Sr$. Methods of absolute β and γ counting have been used for this purpose. The ratio between the two isotopic cross sections is found to be 318 ± 21, from which an activation cross section of 1.59 ± 0.11 b can be calculated for the reaction $^{84}Sr(n, \gamma)\ ^{85}Sr$ (65 d), if the value 5×10^{-3} b is assumed for the cross section of the process $^{88}Sr\ (n, \gamma)^{89}Sr$.

1. Introduction

When irradiated with slow neutrons, a sample containing Sr shows two long-lived activities:

$$^{85}Sr \rightarrow {}^{85}Rb \qquad (65\ d)$$
$$^{89}Sr \rightarrow {}^{89}Y \qquad (50.4\ d)^{1}).$$

The former activity, i.e. ^{85}Sr undergoing a negaton capture process, follows the reaction $^{84}Sr(n, \gamma)^{85}Sr$. The decay scheme of ^{85}Sr has been investigated by several authors[2,3]) whose results agree rather closely: a weakly converted 513 keV γ line follows the capture transition while no similar transition toward the ground state of ^{85}Rb takes place with significant probability. Conversely, ^{89}Sr is produced in the reaction $^{88}Sr(n, \gamma)^{89}Sr$; it is practically a pure β^- emitter. The end point of the negaton spectrum was found [4]) to be 1.462 MeV. Recently evidence has been found that such a β spectrum is not a simple one, because the emission of γ-rays takes place in 2×10^{-4} of the disintegrations [5]). The branching ratio is anyway very weak and does not influence the following considerations to a significant extent.

Absolute measurements of the above-mentioned two activities make it possible to evaluate the ratio of the activation cross sections of the two processes. This has been done by two authors [6,7]) but

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their results do not agree. Consequently it seemed worth while to redetermine the above considered ratio.

2. Experimental Procedure

Two samples of $SrCO_3$, irradiated at different times and with different irradiation times have been supplied by A.E.R.E.

It was preliminarily ascertained that the ratio between the activities of ^{85}Sr and ^{89}Sr was independent of the examined sample when the activities were compared with respect to the same irradiation time and to the same decay interval. A Geiger-Müller electron counter and a scintillation γ-ray spectrometer were used for such a comparison. This circumstance ruled out the possibility that activities due to impurities were present in the samples. Consequently the measurements described below were performed with the more recent activated amount of $SrCO_3$.

γ- or X-ray activity of ^{85}Sr was measured with a sample of Sr which had been electroplated over a thin copper foil; for the measurement of the activity of ^{89}Sr a much thinner source has been used (about 0.1 mg/cm^2 thick) and the ratio between the intensities of the two sources was carefully measured.

3. Absolute Measurements of the Disintegration Rates

a) ACTIVITY OF ^{85}Sr

Three independent methods have been used to measure the activity due to the process

$$^{85}Sr \xrightarrow[65\,d]{} {}^{85}Rb.$$

In a first series of measurements a proportional counter of known efficiency, filled with $A+CH_4$, was used. The pulses due to the emission of K X-rays from K-capture were counted. The disintegration rate D_{85} is proportional to the counting rate N_K according to the following formula:

$$D_{85} = \frac{N_K}{\omega_K \varepsilon_K (1+\alpha_K)} \left(1 + \frac{\lambda_L}{\lambda_K}\right)$$

where ω_K is the K-shell fluorescence yield of Rb ($\omega_K = 0.623$),
α_K the K conversion coefficient of the 513 keV γ-rays of ^{85}Rb ($\alpha_K = 0.008$),
ε_K the efficiency of the proportional counter,

λ_L/λ_K the ratio between the probabilities of L- and K-electron capture ($\lambda_L/\lambda_K = 0.11$)[8]).

X$-\gamma$ coincidences between the above-mentioned proportional counter and a scintillation spectrometer were also used to measure D_{85}. Only the K X-rays detected in the proportional counter and the pulses belonging to the photopeak at the output of the γ-ray spectrometer (crystal of NaI(Tl)) were used to trigger the coincidence circuit. If N_K and N_γ are the counting rates of the two detectors, and $N_{K\gamma}$ is the counting rate at the output of the coincidence circuit, we obtain, obviously,

$$D_{85} = \frac{N_K N_\gamma}{N_{K\gamma}}.$$

A correction had to be made to the obtained value of D_{85} because the resolving time of the coincidence circuit could not be made longer than 3.7μ sec and consequently some γ-rays emitted from the isomeric level of ^{85}Rb ($T_{\frac{1}{2}} = 0.9$ μ sec) were missed by the coincidence apparatus; such a correction amounts to a few percent only.

The γ activity of ^{85}Sr was finally calibrated by comparison with a source of ^{22}Na (positon emitter) whose absolute intensity had been previously measured by means of a coincidence method. For this purpose the counting rates due to the photopeaks at 511 keV (^{22}Na) and 513 keV (^{85}Sr) were compared.

In table 1 the results of the three independent measurements of D_{85} refer to the same instant and their average is given.

TABLE I

Method	D_{85}
Absolute K X-counting with a proportional counter	6.27×10^4 min^{-1}
K X$-\gamma$ coincidences	5.94×10^4 min^{-1}
Comparison with a ^{22}Na source	6.77×10^4 min^{-1}
Average value	$(6.33 \pm 0.16) \times 10^4$ min^{-1}

b) ACTIVITY OF ^{89}Sr

The intensity of K X-rays emitted by a thin source of ^{137}Cs, due to K internal conversion of the well known 662 keV γ line of ^{137}Ba, has been measured by means of the same proportional

counter which had been used to calibrate the sample of $SrCO_3$, and with an entirely analogous method. The source of ^{137}Cs has been successively introduced into an intermediate image magnetic β-ray spectrometer and the area under the peak due to the K and L internal conversion lines has been measured. The ratio of the two counting rates, when corrected for the probability of L conversion in ^{137}Ba, gave directly the transmission of the β-ray spectrometer.

The negaton spectrum emitted from the thin source of $SrCO_3$ has been finally obtained by means of the β-ray spectrometer. The intensity of the β-spectrum was obtained from the area of an ideal Fermi distribution (corrected for the shape factor) taking into account the transmission of the spectrometer.

In this way the absolute number of the negaton emissions taking place within the thin sample of $SrCO_3$ has been determined, and a comparison between this source and the thicker one, which had been used to measure D_{85}, was carried out by measuring their γ activities due to ^{85}Sr, by means of a γ-ray spectrometer and in a standard geometry.

The activity D_{89} of the process

$$^{89}Sr \xrightarrow[50.4\,d]{} {}^{89}Y$$

resulted to be

$$D_{89} = (2.67 \pm 0.16) \times 10^4 \text{ min}^{-1}$$

when referred to the same instant at which D_{85} was measured.

4. Conclusions

The ratio of the activation cross sections of ^{84}Sr and ^{88}Sr is given by

$$\frac{\sigma_{84}}{\sigma_{88}} = \frac{f_{88}}{f_{84}} \varphi(t, T) \frac{D_{85}}{D_{89}} \tag{1}$$

where f_{88} and f_{84} are the abundances of the two considered isotopes in natural strontium,

D_{85}, D_{89} are the disintegration rates of ^{85}Sr and ^{89}Sr,

$\varphi(t, T)$ is a numerical factor accounting for the irradiation time of the sample and for the decays of the two activities until the instant t.

Clearly,

$$\varphi(t, T) = \frac{1 - \exp \lambda_{89} T}{1 - \exp \lambda_{85} T} \exp\left[-(\lambda_{89} - \lambda_{85})t\right]. \tag{2}$$

The numerical values to be introduced into (2) are

$$T = 56\text{d} \qquad t = 89\text{ d}$$
$$\lambda_{89} = 0.0137\text{ d}^{-1} \qquad \lambda_{85} = 0.0107\text{ d}^{-1}$$

so that we obtain $\varphi(89, 56) = 0.910$.

If f_{88}/f_{84} is taken to be 147.4, and considering the above quoted values of D_{85} and D_{89}, we have $\sigma_{84}/\sigma_{88} = 318 \pm 21$.
When σ_{88} is assumed to be [9]) 5×10^{-3} b, the resulting value for the activation cross section of ^{84}Sr is

$$\sigma_{84} = 1.59 \pm 0.11\text{ b}$$

where the quoted error simply represents the uncertainty of our measurement.

Such a value is to be compared with the values given by Harrison and Seymour [6]) and by Lyon [7]). This is done in Table II.

TABLE II

Author	σ_{84}
G. E. Harrison and F. D. Seymour [6])	0.32±0.06 b
W. S. Lyon [7])	1.2 ±0.1 b
Present value	1.59±0.11 b

While the agreement with the value given by Lyon cannot be considered unsatisfactory, the discrepancy between the result of Harrison and Seymour and ours is well beyond the uncertainties usually involved in absolute calibrations and cannot be easily explained.

We are indebted to prof. G. Bolla for his interest in the present measurement.

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BETA DECAY OF ^{161}Tb AND LOW-LYING LEVELS IN ^{161}Dy

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Received 6 February 1956

Abstract: The β^- and γ-radiations from ^{161}Tb were analysed by means of a scintillation spectrometer, a proportional counter, a β-ray spectrometer and $\gamma-\gamma$ and $\beta-\gamma$ coincidence techniques. γ-rays were observed at energies 25.9 ± 0.2, 49 ± 2, 77 ± 2 keV. The end-point of the β^- spectrum occurs at 500 ± 3 keV. The half-life was found to be $T_{\frac{1}{2}} = 6.9\pm0.1$ d. A decay scheme is proposed.

1. Introduction

The activity of ^{161}Tb ($T_{\frac{1}{2}} = 6.8$ d) has been observed by several investigators [1]). While the existence of the emission of β^- particles (0.5 MeV) has been well established by means of absorption measurements, there is no conclusive information on γ-radiations. This fact can be noted from Table I where previous results are summarized.

The Coulomb excitation with α particles of low-lying (rotational) levels of natural Dy was observed by Heydenburg and Temmer [2]). Two γ-rays, at 76 keV and 166 keV energies, were found. The 76 keV γ-ray was assumed to belong to both even isotopes (^{162}Dy and ^{164}Dy) and odd neutron isotopes (^{161}Dy and ^{163}Dy). The 166 keV γ-ray was tentatively attributed to one or both odd neutron isotopes. These γ-rays were interpreted as coming respectively from the first and the second rotational excited states. As can be seen from Table I these γ-rays were not known previously.

Consequently it seemed worthwhile to reinvestigate the radiations from ^{161}Tb with the purpose both of clarifying the decay and comparing the data obtained by means of β- and γ-ray spectroscopy with those obtained from Coulomb excitation. In effect the information concerning the low-lying levels of nuclei with $A > 140$ is interesting for the checking of the unified model of Bohr and Mottelson. This is true for odd mass nuclei where experimental data are rare.

TABLE I

Half-life d	Energy of radiation in MeV		References
	β^-	γ	
7	0.5 abs	0.05 abs	Ketelle [3])
6.75	0.52 abs	0.05 abs	Butement [4])
		0.026 ion ch	Scharff-Goldhaber *et al.*[5])
7.2	0.50 abs		Hein and Voigt [6])
6.8		0.049 (L/M = 3.7) spect conv	Cork *et al.* [7])
		0.0488 cryst. spect	Jaffe [8])
	0.550 spect	0.046–0.049, 0.075 scint spect, $\beta-\gamma$, $\gamma-\gamma$ coinc	Barloutaud and Ballini [9])

The adopted symbols and abbreviations are taken from Hollander, Perlman and Seaborg [1]).

2. Measuring Techniques

The sample of ^{161}Tb was obtained as daughter of ^{161}Gd ($T_{\frac{1}{2}}$=3.7m) by neutron irradiation of spectroscopically pure gadolinium oxide in the Harwell pile. The specific activity was about 1 mC/g. After the short-living activities were allowed to decay, the sample emitted radiations from ^{161}Tb and ^{153}Gd, the latter of which decays through orbital electron capture ($T_{\frac{1}{2}}$ = 236 d).

The γ- and X-radiations were analysed using a single crystal scintillation spectrometer and a proportional counter spectrometer. The pulse size was measured by means of a twenty-channel electronic pulse analyser [10]).

A high transmission intermediate image β-ray spetrometer was used for investigating the β^- spectrum. The baffles in the spectrometer were adjusted for a resolving power of 4 per cent.

$\gamma-\gamma$ and X$-\gamma$ coincidence technique was used both between the proportional counter and the scintillation spectrometer, and between the two scintillation spectrometers. Only the pulses belonging to the photopeak at the output of the γ-ray spectrometers were used to trigger the coincidence circuit. $\beta-\gamma$ coincidences were also obtained using a hollow crystal spectrometer for β detection. The resolving time of the coincidence circuit was about 3.5 μsec.

3. Radiations from the Sample; Half-life

3.1 γ SPECTRUM

The inspection of the γ spectrum (fig. 1 and fig. 2), in the scintillation and in the proportional counter spectrometer, showed the presence of peaks at the following energies: 25.9 ± 0.2 keV,

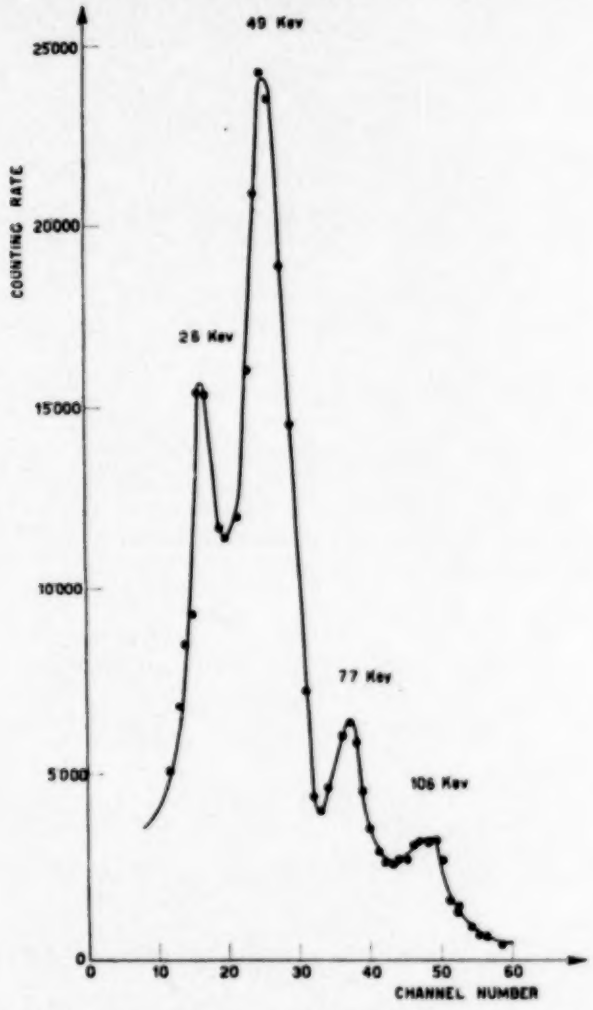

Fig. 1. γ-ray spectrum (scintillation spectrometer).

49 ± 2 keV (composite peak), 77 ± 2 keV, 106 ± 2 keV. When the activity of ^{161}Tb practically ceased, only two longliving peaks at 41.5 keV (K X-rays characteristic of Eu) and 106 keV energies were noticed, which was to be expected owing to the activity [1]) of ^{153}Gd.

3.2 β^- SPECTRUM

The β sample was prepared as follows. The terbium oxide target material was dissolved in HCl, then the accurately-dried terbium

chloride was dissolved in pyridine. Cu foils ($\approx$ 0.5 mg/cm^2) were electroplated in the solution using inox steel rotating anode. The sample thickness was 0.05 mg/cm^2.

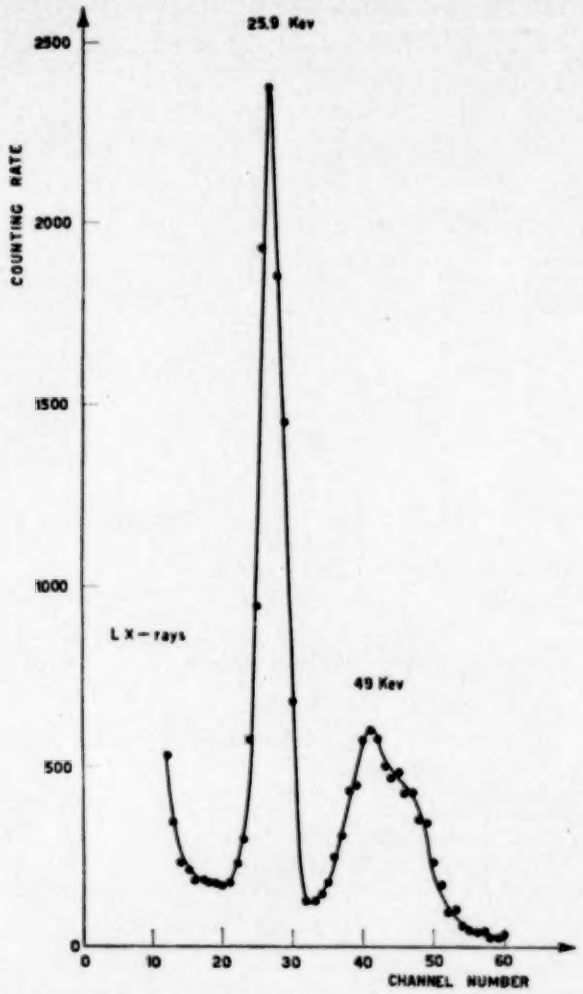

Fig. 2. γ-ray spectrum (proportional counter spectrometer).

The Fermi plot of the obtained spectrum is shown in fig. 3. The plot is substantially a straight line down to about 100 keV; the end-point was found to be

$$E_0 = 500 \pm 3 \text{ keV}.$$

For energies lower than 100 keV the β spectrum is distorted by a complex group of internal conversion lines and Auger lines.

3.3 COINCIDENCE MEASUREMENTS

Coincidence measurements showed that

a) the 25.9 keV peak is in coincidence with the 49 keV peak;

b) when the composite peak at 49 keV is used to trigger the coincidence circuit, the appearance of the 25.9 keV peak, Eu K X-rays peak, and the 106 keV peak is easily revealed;
c) by triggering the coincidence circuit with the 77 keV peak, no evidence of peaks was found;
d) by triggering the coincidence circuit with the 106 keV peak, only the K X-rays of Eu appeared;
e) when the coincidence circuit was then triggered by the β^- spectrum in the energy interval between 200 and 500 keV, the 25.9 keV, 49 keV and 77 keV peaks were found coincident with the β-rays.

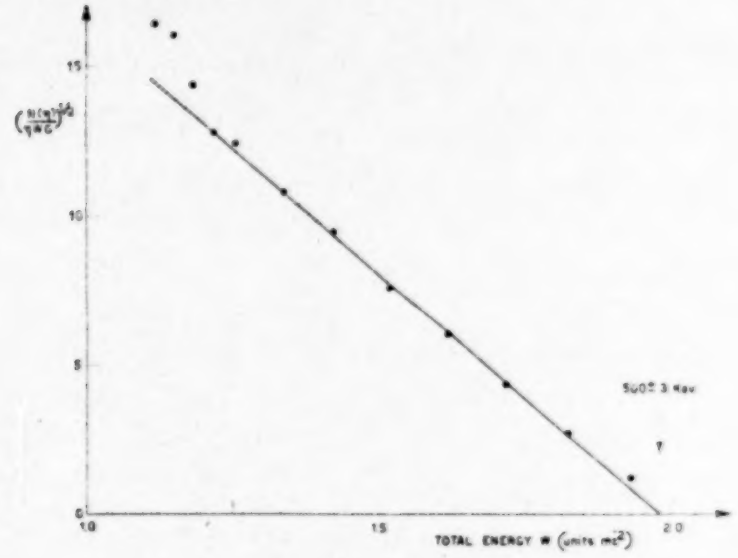

Fig. 3. Fermi plot of the β^- spectrum.

3.4 HALF-LIVES

The half-lives of the above radiations were measured following the time decrease of the area under the peaks over about 40 days. The values obtained are listed in Table II.

TABLE II

	Energy of radiations from ^{161}Tb (keV)	$T_{\frac{1}{2}}$ (d)	Relative intensity
γ_1	25.9±0.2	6.9±0.1	1
γ_2+K X (composite peak)	49 ±2	6.5±0.5	1.82
γ_3	77 ±2	6.7±0.3	0.57
β^-	500 ±3	6.8±0.3	≈10

On the basis of the coincidence measurements and of the half-lives we conclude that the γ-rays listed in Table II arise from the β^- decay of ^{161}Tb. For the half-life we adopt the value

$$T_{\frac{1}{2}} = 6.9 \pm 0.1 \text{ d}.$$

4. Intensity Measurements

The intensity measurements were performed in the conventional manner. The obtained values, corrected for detection efficiencies, escape peaks, and, when necessary, for the activity of ^{153}Gd, are listed in Table II. In order to deduce the decay scheme, a rough estimate of the conversion coefficient is needed. The following equations may be written:

$$\frac{I_{\gamma 2}}{I_{\gamma 3}} + \alpha_{K,3}\,\omega_K = 3.2 \qquad (1)$$

$$\frac{I_{\gamma 1}}{I_{\gamma 2} + \alpha_{K,3}\omega_K I_{\gamma 3}} = 0.55 \qquad (2)$$

where $I_{\gamma i}$ is the intensity of the γ_i-rays,
ω_K the K-shell fluorescence yield of Dy,
$\alpha_{K,3}$ the K conversion coefficient of the γ_3-rays (the K conversion of the γ_1- and γ_2-rays is energetically impossible).

From eq. (1) and considering that [11]) $\omega_K = 0.92$, it can be deduced that

$$\alpha_{K,3} < 3.5.$$

An extrapolation of the theoretical values of the internal conversion coefficient for the K-shell [12]) gives for electric quadrupole (α_2) and magnetic dipole (β_1) at 77 keV

$$\alpha_2 \approx 2.9, \qquad \beta_1 \approx 4.8.$$

Consequently we assume that the 77 keV γ-rays arise from an electric quadrupole transition. That is quite reasonable if the 77 keV γ transition is the same as that observed by Temmer and Heydenburg in Coulomb excitation of Dy; in fact the excitation processes are limited to E2 transitions.

An estimate of the total conversion coefficient ($\alpha_{\gamma 2}$) of the 49 keV γ-rays was made using coincidence measurements (sec. 3.3). It was found that

$$\alpha_{\gamma 2} \approx 1.5.$$

This result suggests that the 49 keV γ-rays arise from a magnetic dipole transition [12]).

By adopting the theoretical value for $\alpha_{K,3}$, eqs. (1) and (2) give

$$\frac{I_{\gamma 2}}{I_{\gamma 3}} = 0.54, \tag{3}$$

$$\frac{I_{\gamma 1}}{I_{\gamma 3}} = 1.75. \tag{4}$$

Consequently

$$\frac{I_{\gamma 1}}{I_{\gamma 2}} = 3.2. \tag{5}$$

Under the reasonable assumption that $\alpha_{\gamma 2} < \alpha_{\gamma 1}$, it can be deduced from eq. (5) that the 26 keV transition occurs with greater intensity than the 49 keV transition.

5. Decay Scheme and Discussion

The results of the preceding section suggest a decay scheme of ^{161}Tb as shown in fig. 4.

Spin and parity of the ground level of ^{161}Dy are known [1])[13]); the orbital of the ground state of ^{161}Tb has been assigned on the basis of the nuclear shell model. The parities of the excited levels are suggested by the preceding measurements; no concrete assignment of the spins can be made.

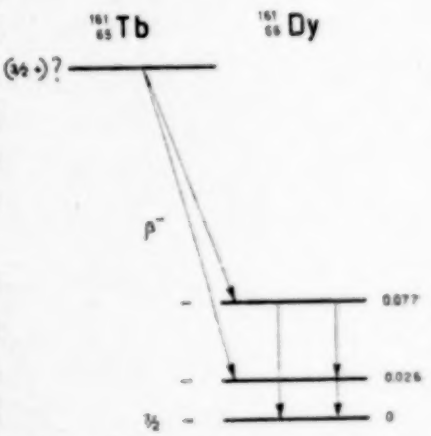

Fig. 4. Proposed decay scheme.

As regards a β^- branching to the ground state of ^{161}Dy, the transition should be classified: $\Delta I = 2$, yes. In fact, from an estimate of the total intensity of the β^--rays relative to the intensity of the γ-rays, no significant branching to the ground state of ^{161}Dy seems to occur.

As regards the β^- branching to the 77 keV level of ^{161}Dy, a rough estimate of the branching ratio shows that it should be

$$6.5 < \log ft < 8.4.$$

Consequently the transition should be classified, according to Mayer, Moszkowski and Nordheim [14]), as first forbidden ($\Delta I = 0, 1$; yes).

As previously said, the 77 keV level is probably the first rotational level according to Heydenburg and Temmer [2]); then its orbital should be $9/2^-$. This fact suggests that the assignment of the spin to the ground state of ^{161}Tb (on the basis of the nuclear shell model) and to the 77 keV excited level of ^{161}Dy (on the basis of the unified model of Bohr and Mottelson) is inconsistent with the existence of the above-mentioned β^- branching.

Our thanks are due to prof. G. Bolla for his kind interest.

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POLARISATION OF HIGH ENERGY NUCLEONS SCATTERED BY NUCLEI

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Received 13 February 1956

Abstract: Calculations have been made on the polarisation of protons scattered by nuclei at 135 and around 300 MeV. With a spinorbit coupling similar to that used by E. Fermi good agreement with experiments made at Harwell and Berkeley is obtained for medium and heavy elements while not for the lightest. An explicit formula for the polarisation of neutrons is derived in certain approximations and discussed.

1. Introduction

Experimental evidence for the polarisation of protons through scattering was first reported from the University of Rochester. Reports of calculations on the polarisation of protons scattered elastically by nuclei have been given in previous publications from this institute [1,2]). Further measurements have become available and calculations have been made that are presented here.

As in the earlier calculations I have tried to represent the nucleon-nucleus interaction by a spherically symmetric potential

$$V = V(r)(1+i\beta)+\alpha\frac{V'(r)}{r}\mathbf{l}\cdot\mathbf{s} \qquad (1)$$

with

$$\alpha = -\alpha_1\cdot\frac{\hbar^2}{2M^2c^2}, \qquad \alpha_1 = 30.$$

Here $\mathbf{l}$ and $\mathbf{s}$ are the spin and orbital angular momenta in units of $\hbar$ and M is the mass of the scattered nucleon; β is a constant.

With $V(r)$ representing a square well the first part of (1) is the potential introduced by S. Fernbach, R. Serber and T. B. Taylor [3]) to describe high energy scattering cross-sections. The second, the spin-orbit coupling, was suggested (including the value of α_1) by E. Fermi [4]) to account for polarisation experiments.

In this paper the potential (1) will only be regarded as a phenomenological interaction. It remains to be seen how far it is possible

to adjust the radial dependence of the central potential $V(r)$ and the parameters in (1) to get agreement with the experiments. Eventually one will also have to let α_1 be a function of the nuclear mass and of the energy of the interacting nucleon.

The calculations in this paper have been performed in the same way as in ref. [2]). The phase shifts have been calculated in Born approximation and the depths of the real and imaginary parts of the central potential (assumed to be functions of energy only) were adjusted to agree with experimental absorption (σ_a) and total (σ_t) cross sections [5,6]), when these are calculated for neutrons. The nuclear part of the interaction was assumed to be the same for neutrons and protons. In the calculations on protons the Coulomb potential was added to the central potential; the protons in the nucleus were supposed to be uniformly distributed within a radius $r_0 A^{1/3}$, with [7]) $r_0 = 1.2 \times 10^{-13}$ cm (the precise figure is not important for these calculations).

Most of the calculations in this paper, again like in ref. [2]) have been made with a central potential of parabolic shape:

$$V(r) = V_0\left(1-\frac{r^2}{R^2}\right) \qquad (r \leqq R)$$

$$V(r) = 0 \qquad (r > R)$$

with [8]) $R = r_1 A^{1/3}$, $r_1 = 1.6 \times 10^{-13}$ cm. It was found that the radius used did not reproduce cross-sections for heavy nuclei using the depths determined from cross-sections for light nuclei at the same energy. Some ambiguity exists, however, anyway, in the choice of depths for protons when they are determined for neutrons. The major effect of a change in radius (keeping the depths constant) is to expand or contract the angular patterns of the polarisation curve. The effect of a change in the depths of the wells (connected with a change in radius through the condition to reproduce total and absorption cross-sections) has been investigated.

The Born approximation for the phase shifts is assumed to be applicable to small-angle scattering [9,15]). We shall therefore, in this paper, make comparisons only with experiments for angles smaller than or not much larger than that of the first diffraction minimum. In connection with this discussion see the comparison with the W.K.B. approximation made in ref. [2]).

2. Results of Calculations

In the region of medium and heavy nuclei calculations have been made on Fe, Ag and Bi at 135 MeV and on Ca at 310 MeV. The

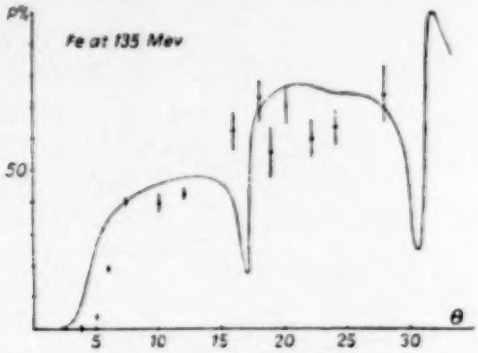

Fig. 1. $_{26}Fe^{56}$ at 135 MeV. Experimental points from Harwell.
$V_0 = -26$ MeV, $\beta = 1.00$, $\sigma_a = 0.76$, $\sigma_t = 1.43$.
Exp.: $\sigma_a = 0.74$, $\sigma_t = 1.41$.

results of these and comparison with experiments [10,11]) are given in fig 1—4. The upper curves show the differential cross-sections,

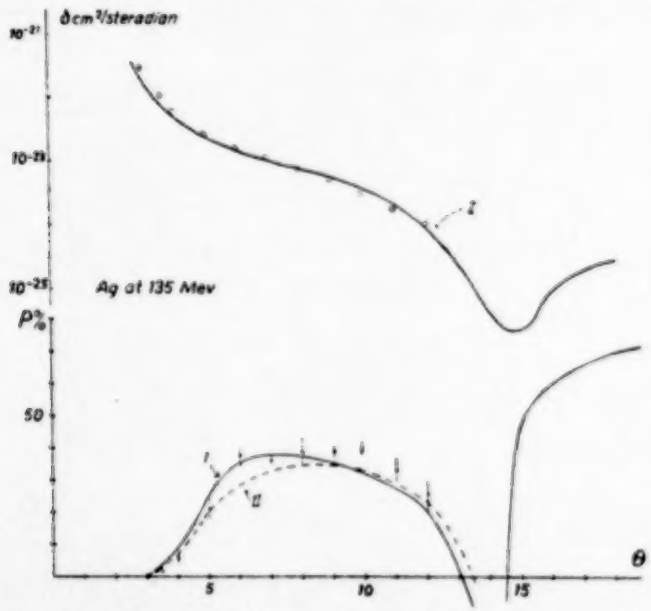

Fig. 2. $_{47}Ag^{108}$ at 135 MeV. Experimental points from Harwell.
Curve I: $V_0 = -27$ MeV, $\beta = 0.74$, $\sigma_a = 1.16$, $\sigma_t = 2.40$.
II: $V_0 = -26$ MeV, $\beta = 1.00$, $\sigma_a = 1.26$, $\sigma_t = 2.43$.
Exp.: $\sigma_a = 1.16$, $\sigma_t = 2.40$.

the lower the polarisation. The angle θ is given in the laboratory system. Corrections for the finite mass of the target nucleus are made in the case of Ca and Ag. One should note that for Fe at

135 MeV the calculated minimum in the polarisation at about 17° is not present in the experiments. In the calculations on Ag and Ca the corresponding minima appear at around 14° and are pronounced in the experiments. Also measurements at 315 MeV on Fe show this minimum [11]). It is therefore believed that the minimum for Fe at 135 MeV also does exist and will subsequently be found if looked for. We conclude that the calculations reproduce the characteristic features of the experiments and they also give the

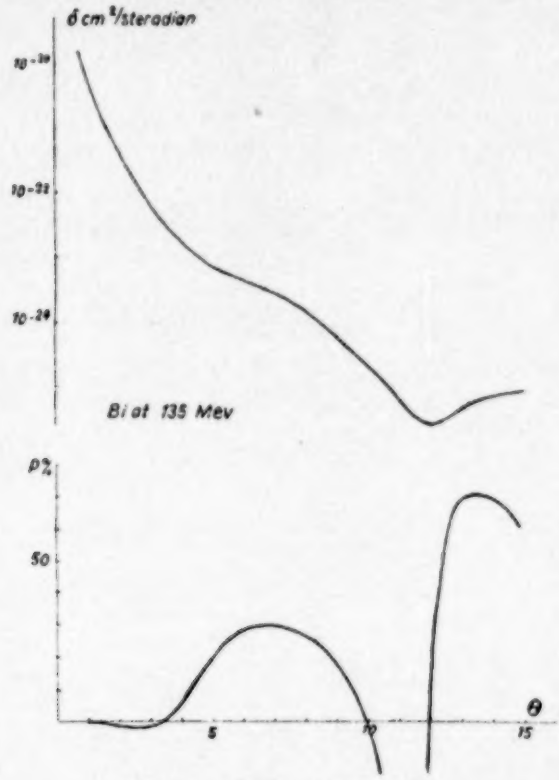

Fig. 3. $_{83}Bi^{209}$ at 135 MeV.
$V_0 = -27$ MeV, $\beta = 0{,}74$, $\sigma_a = 1.93$, $\sigma_t = 4.05$.
Exp.: $\sigma_a = 1.76$, $\sigma_t = 3.92$.

right magnitude of the polarisation within or not much outside the experimental errors.

It should be observed that the same value of $\alpha_1 = 30$ has been used in all calculations and also that $\alpha_1 = 30$ was found in an estimate of Heisenberg [12]) on the splitting of levels due to spin-orbit interaction in the shell-model. For energies in the region below 135 MeV the known experimental material is still too poor to allow an estimate of α_1 and it is not believed that we are justified to draw any final conclusions from the calculations of ref. [1, 2, 13]) although these indicate a smaller α_1 at 90 and 50 MeV.

For neutrons it has been found (see eq. (6) below) that the magnitude of the polarisation is approximately proportional to the product $\alpha_1\beta$, where β defines the imaginary potential (see eq. (1)). Thus an uncertainty in β will lead to an uncertainty in α_1 if agreement with experiments is to be preserved. To see what the situation is like for protons as regards this point calculations have been made on Ag with $\beta = 0.74$ and $\beta = 1.0$ as shown in fig. 2. One sees how through the Coulomb interference the polarisation is somewhat decreased around 6^0 when β is increased. For

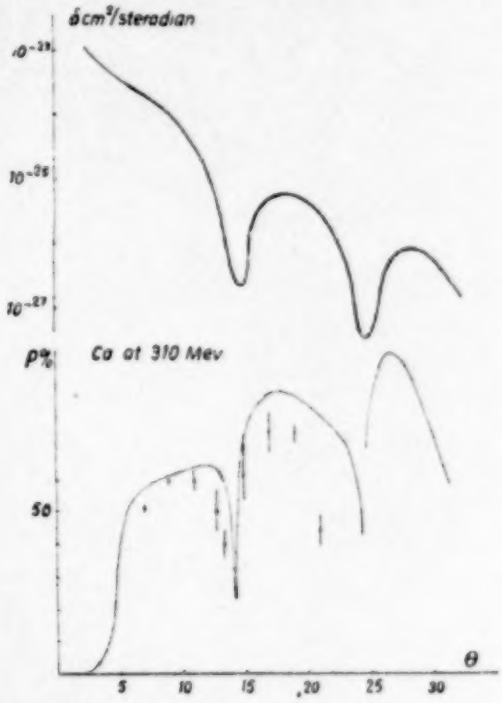

Fig. 4. $_{20}Ca^{40}$ at 310 MeV. Experimental points from Berkeley.
$V_0 = -24$ MeV, $\beta = 1.25$, $\sigma_a = 0.51$, $\sigma_t = 0.82$.
Exp.: $\sigma_a = 0.50$, $\sigma_t = 0.82$.

neutrons on the other hand we should expect an increase by 1/3. We thus see that the effect of the Coulomb field is such that α_1 is better determined at least in this case. The more detailed investigation on carbon mentioned later in this paper gives for the polarisation of protons a similar dependence on β as was found in the case of Ag and also in the case of Bi in a calculation not presented in this paper. For other energies the behaviour may be different.

For lighter nuclei calculations have been made on C and Al at 135 MeV and on C at 289 MeV (fig. 5—7). Corrections for the finite mass of the target nucleus were made in all cases except

for curves I and II of fig. 5. We observe that these calculated polarisation curves do not reproduce the characteristic features of the experimental curves as was found for the heavier elements. At 135 MeV experiments show a minimum in cross-sections at an angle where nuclear and Coulomb scattering are comparable and that is especially pronounced for carbon. To see how this is connected with the form chosen for the central well a number of calculations were made. It was found that if the radius of the parabolic well is

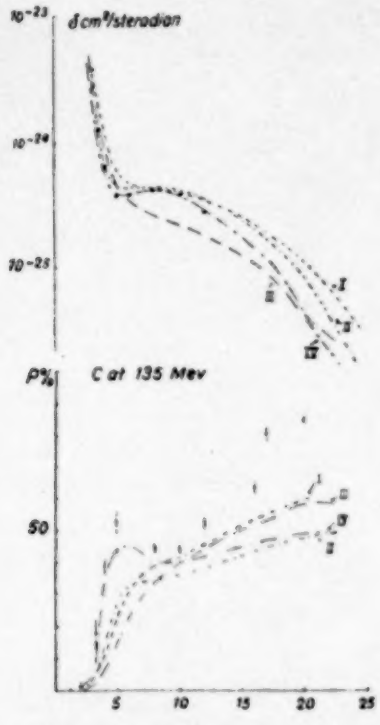

Fig. 5. ${}_6C^{12}$ at 135 MeV. Experimental points from Harwell.
Curve I: Parabolic pot. $r_1 = 1.6$, $V_0 = -26$ MeV, $\beta = 1.00$, $\sigma_a = 0.22$, $\sigma_t = 0.38$,
II: Parabolic pot. $r_1 = 1.8$, $V_0 = -17$ MeV, $\beta = 0.76$, $\sigma_a = 0.23$, $\sigma_t = 0.39$.
III: Gaussian pot. $r_2 = 1.0$, $V_0 = -20$ MeV, $\beta = 1.40$. $\sigma_a = 0.23$, $\sigma_t = 0.37$.
IV: Gaussian pot. $r_2 = 1.2$, $V_0 = -24$ MeV. $\beta = 0.58$, $\sigma_a = 0.23$, $\sigma_t = 0.37$.
(r_1, r_2 in units 10^{-13} cm) Exp.: $\sigma_a = 0.23$, $\sigma_t = 0.37$.

increased the interference effect is increased (curves I, II). For a square well an increase of radius did not have such an effect. Associated with an increase in radius a decrease in β is necessary to reproduce total cross-sections. As expected (see e.g. eq. (6)) this gives a decrease of polarisation for angles where Coulomb scattering is unimportant. It is to be expected that especially for light nuclei it should be important to take account of the diffuseness of the surface. Therefore calculations were also made

with a gaussian central well $V(r) = V_0 \cdot e^{-r^2/R^2}$ with $R = r_2 \cdot A^{1/3}$ and $r_2 = 1.0$ and 1.2 in units 10^{-13} cm. The result of these was, however, quite similar to that obtained for a parabolic well (curves III, IV) †.

Further it was found that calculations of polarisation on carbon in which the real part of the parabolic central well was kept constant and β was changed gave the same dependence on β as was obtained indirectly when the radius was changed. So it seems as if the form

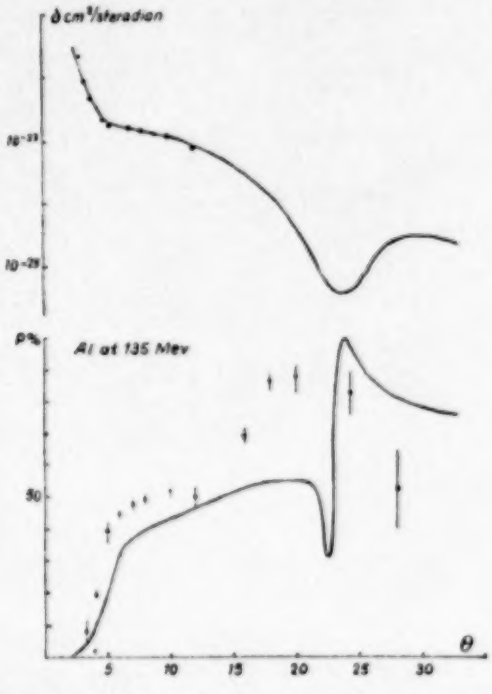

Fig. 6. $_{13}Al^{27}$ at 135 MeV. Experimental points from Harwell.
$V_0 = -26$ MeV, $\beta = 1.00$, $\sigma_a = 0.43$, $\sigma_t = 0.77$.
Exp.: $\sigma_a = 0.43$, $\sigma_t = 0.79$.

of the calculated curve (in this angular region) is determined mainly by β and not so much by the form of the central well. The calculations mentioned above should be compared with those on Ag, and with fig. 3 of ref. [13]).

It should also be mentioned that calculations have been made on carbon at 135 MeV with the absorption concentrated either at the centre or at the surface of the nucleus, while the real part of the potential was kept parabolic with $r_1 = 1.6 \times 10^{-13}$ cm. In the

† It should be observed that the β of curve IV is smaller than that of the curve II. It may be significant that the curves IV agree well with experiments for small angles. (Observe, however, that absolute values of experimental cross-sections are not given.)

first case the curve was like I and in the second an oscillating curve with a minimum of −85 % at 30° was obtained for the polarisation.

The calculation on carbon at 289 MeV (fig. 7) gives good agreement with experiments for small angles. For larger angles the disagreement is evident. It is not expected that other forms of the central potential will give better agreement.

We have noticed how the interaction (1) is capable of describing

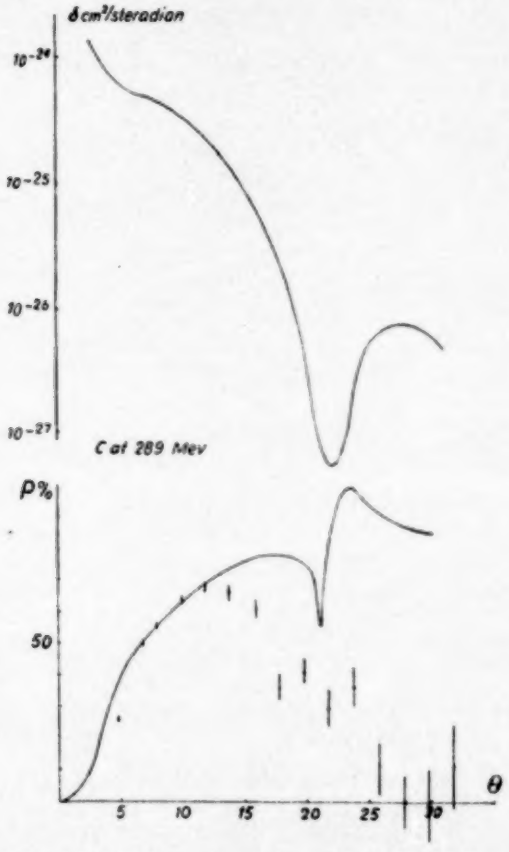

Fig. 7. ${}_6C^{12}$ at 289 MeV. Experimental points from Berkeley.
$V_0 = -21$ MeV, $\beta = 1.29$, $\sigma_a = 0.20$, $\sigma_t = 0.30$.
Exp.: $\sigma_a = 0.20$, $\sigma_t = 0.28$.

the polarisation experiments for medium and heavy nuclei fairly well. For the lightest nuclei it seems more difficult to get agreement with experiments. The important difference between heavy and light nuclei in this connection should be that light nuclei present a larger fraction of the nucleons at the surface (the fraction varying as $A^{-\frac{1}{3}}$). This may not be taken care of altogether by making the potential smooth at the edge, although it seems to explain the

absence of diffraction minima for light nuclei [11,13]). Other surface effects, e.g. scattering on practically free nucleons at the surface, are perhaps important.

It should be pointed out that all calculations presented in this paper (and all published so far on this subject) are made on spherically symmetric wells. Actual nuclei except those with closed shells are known to be deformed. We observe that most of the nuclei on which calculations have been made in this paper are not far from closed shell configurations. However the non-symmetry of nuclei will perhaps have to be considered before final conclusions regarding the usefulness of the interaction (1) can be drawn.

3. A General Discussion

In this paper a parabolic central potential has been used mostly, although that shape is not expected to give a good picture of actual nuclei in all cases. For a discussion of how the form of the central potential affects the results on the calculations of the polarisation it is of some help to consider the explicit formula for the polarisation of neutrons given in ref. [2]) and its derivation will now be presented.

The polarisation, as usually defined, is given by

$$P(\theta) = \frac{2\,\mathrm{Im}\,A^*B}{|A|^2+|B|^2}, \tag{2}$$

where †

$$\begin{aligned} A(\theta) &= \sum_{l=0}^{\infty} [(l+1)\exp(2i\delta_{l+\frac{1}{2}}) + l\cdot\exp(2i\delta_{l-\frac{1}{2}}) - (2l+1)]P_l(\cos\theta), \\ B(\theta) &= \sum_{l=0}^{\infty} [\exp(2i\delta_{l+\frac{1}{2}}) - \exp(2i\delta_{l-\frac{1}{2}})]P_l^1(\cos\theta). \end{aligned} \tag{3}$$

If the scattered particle is uncharged and the interaction is given by (1) the phase shifts are in Born approximation

$$\begin{aligned} \delta_{l+\frac{1}{2}} &= \delta_l + \frac{l}{2}\,\delta_l^s \\ \delta_{l-\frac{1}{2}} &= \delta_l - \frac{l+1}{2}\delta_l^s \end{aligned} \quad \text{with} \quad \begin{cases} \delta_l = -\dfrac{1+i\beta}{2E}\displaystyle\int_0^\infty V(r)\,dz \\ \delta_l^s = -\dfrac{\alpha}{2E}\displaystyle\int_0^\infty \frac{V'(r)}{r}\,dz, \end{cases} \tag{4}$$

where ††

$$z = [k^2r^2 - (l+\tfrac{1}{2})^2]^{\frac{1}{2}}.$$

† P_l and P_l^1 are defined as in Schiff, Quantum Mechanics, p. 71.

†† As in ref. [2]), formula (2), $l+\frac{1}{2}$ should be replaced by l for $l=0$. That is, however, unimportant for the considerations made here.

One finds

$$(l+\tfrac{1}{2})\delta_l^s = \frac{\alpha k^2}{1+i\beta}\frac{\mathrm{d}}{\mathrm{d}l}\delta_l; \tag{5}$$

δ_l is a monotonously decreasing function of l. From (5) one finds that for a square well δ_l^s becomes infinitely large for $l = kR-\frac{1}{2}$; but if the edge of the well is smoothed out this infinity vanishes.

We now treat $(l+\frac{1}{2})\delta_l^s$ as small by putting $\exp(il\delta_l^s) = 1+il\delta_l^s$, and get

$$\begin{cases} A(\theta) = \sum_{l=0}^{\infty}(2l+1)(\exp(2i\delta_l)-1)P_l(\cos\theta), \\ B(\theta) = \sum_{l=0}^{\infty}\frac{\alpha k^2}{1+i\beta}\exp(2i\delta_l)\frac{\mathrm{d}}{\mathrm{d}l}(2i\delta_l)P_l^1(\cos\theta). \end{cases}$$

Replacing $\exp(2i\delta_l)\mathrm{d}/\mathrm{d}l(2i\delta_l)$ by $\frac{1}{2}(\exp(2i\delta_{l+1})-\exp(2i\delta_{l-1}))$ and using the relation $(2l+1)P_l = (\sin\theta)^{-1}(P_{l+1}^1 - P_{l-1}^1)$ one gets

$$B(\theta) = -\frac{\alpha k^2}{2(1+i\beta)}\sin\theta\cdot A(\theta)$$

and

$$P(\theta) = \frac{\alpha\beta k^2\sin\theta}{1+\beta^2+\frac{1}{4}\alpha^2k^4\sin^2\theta}. \tag{6}$$

In the above derivation the phases δ_l have not been treated as small as has been done in ref. [14, 15]) in deriving the same formula (6).

One notes that the simple formula (6) is obtained because in the approximations made the sums $A(\theta)$ and $B(\theta)$ have the same dependence on scattering angle except for a factor $\sin\theta$. Eq. (6) will be a bad approximation for those angles for which $A(\theta)$ and $B(\theta)$ are small (around the diffraction minima) and the polarisation is sensitive to small errors in the phase-shifts. This detail is also discussed in ref. [11]) p. 14. One further notes that eq. (6) does not depend directly on what radial dependence is used for the central potential †. (Indirectly it does through β).

The essential approximation made was to treat the spin-orbit phase-shifts as small. To get an estimate of how these vary with energy, nuclear radius and potential we put simply

† This result is a little discouraging, as in an early stage of these calculations it was hoped that polarisation experiments would be a sensitive tool in an ultimate study of the density distribution in nuclei. It should perhaps be pointed out that the independence of the form of the central well in (6) is a consequence of the specific form of the spin-orbit coupling assumed in (1).

$$\frac{\mathrm{d}}{\mathrm{d}l}\delta_l = \frac{\delta_0 - \delta_{kR}}{k \cdot R} = \frac{V_0(1+i\beta)}{2E},$$

where V_0 is an average depth of the potential. Thus

$$(l+\tfrac{1}{2})\delta_l^s = -\frac{V_0 \cdot \alpha_1}{2Mc^2} \tag{7}$$

and we find that the magnitude of $(l+\frac{1}{2})\delta_l^s$ is determined mainly by V_0 and α_1. The depth of the central potential increases markedly below 130 MeV (see e.g. ref. [16])). So we expect eq. (6) to be a worse approximation for energies $\ll$130 MeV, if we suppose α_1 to be a constant. (Also we then expect the Born approximation for the phase-shifts to be worse.) The correction terms to (6) should depend on the radial dependence of the central potential. For calculations of the polarisation at lower energies one can therefore suspect an increased dependence on the radial form of the well (supposing again α_1 to be a constant). This point was also discussed in ref. [2]).

For the scattering of protons the Coulomb field enters and will be important for small angles where Coulomb scattering is dominant and also for angles where Coulomb and nuclear scattering are comparable, giving rise to interference effects in certain cases (cf. calculations on C) and finally also around the diffraction minima †. One is therefore led to suppose that the calculated polarisation for protons should depend more sensitively on what radial form is chosen for the central well than it does for neutrons. The actual calculations for energies $\geqq$ 130 MeV show that this dependence in general is rather small ††. (Cf. e.g. the calculations in ref. [1, 2, 13]). Therefore it is believed that the choice of a more physical well than the parabolic central potential does not affect the results of the calculations of the polarisations to the extent of invalidating the conclusions drawn in this paper.

† A calculation on Cu at 135 MeV, in which $A(\theta)$ was separated into a neutron-like sum and another one that took care of the influence of the Coulomb field, showed the latter sum to have a maximum at an angle just smaller than the angle of the diffraction minimum.

†† Note, however, the difference in the form of the polarisation minimum in ref. [13]), fig 3 as compared to ref. [1]), fig. 2 and fig. 1 in this paper. This suggests that from very careful measurements (and careful calculations) of elastic scattering around the minimum one can learn about the form of the central well. (Note that this is in accordance with the discussion above.)

I wish to express my gratitude to Professor Ivar Waller for suggesting this investigation and for many valuable discussions. I also want to thank Fil. mag. T. Eriksson and Dr. W. J. Swiatecki for some interesting discussions, and Fil. mag. Vollrad Lundström who has made some of the numerical computations.

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ON THE UNIVERSAL INTERACTIONS †

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Received 9 January 1956

Abstract: The possibility of explaining the complicated interactions between elementary particles in terms of universal interactions is discussed. Four types of interactions which seem to have a certain universal character are mentioned in section 1. They are 1.1. electromagnetic interaction (*e*-int.), 1.2. strong baryon-meson interaction (*G*-int.), 1.3. weak Fermi interaction (*f*-int.) and 1.4. weak boson-fermion interaction (*g*-int.). The possibility of having the unique coupling constant *g* for the case 1.4 is investigated by the analysis of decays of hyperons and K_μ-meson allowing their spin values to be up to 3/2 spin and 1 respectively (§ 2). The best fit is found in the derivative couplings between spin 1/2 fermions and spin 0 bosons, but the possibility of including higher spin values is not completely ruled out. From a speculative multiple particle theory of *g*-int., the space inversion parity of elementary particles is determined from their decay modes. With the object of decreasing the number of universal interactions, the elimination of *g*-int. through the other three possible universal ones is discussed in § 3. It is shown, however, that this cannot be possible without adding a more complicated structure to *f*-int. Thus, merely by considering the ratio of $\pi \to \mu+\nu$ and $\pi \to e+\nu$ decay, it is concluded that (i) the coupling types of *f*-int. are process dependent, as long as we want the same value of the coupling constant for *f*-int. The possible coupling types and their experimental tests are investigated. Moreover, it is shown that, in order to explain hyperon decays (ii) *f*-int. must be extended to four baryon (not neutrino) processes. Under the conditions (i) and (ii) there seems to remain, at present, still the possibility that *g*-reactions such as hyperon decays and $K_\mu \to \mu+\nu$ etc. can be explained by such an intermediation, if (iii) the correct $\pi \to \mu+\nu$ decay life-time is given by the same mechanism. If any of the conditions (i), (ii) and (iii) does not hold, the coexistence of the elementary *g*-int. is the simplest and the most natural assumption. It is also shown that by the intermediary of these universal interactions, the interesting K-meson-decay modes $K_{2\pi} \to \pi+\pi^0$ and $K_3 \to \mu$ (or e)$+\nu+\pi^0$, can be the alternative decay modes of the same K-meson (§ 4 and Appendix II). In these discussions the perturbation treatments with regard to *G*-int. are

† A preliminary account of the present paper was given in "Soryushiron Kenkyu" (Mimeographical circular in Japanese) **9** (1955) 559.

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avoided as much as possible. In § 5 the elementarity of these interactions is briefly summarised and the existence of other sorts of interactions is referred to in connection with the existence of the $\varkappa$-meson. In Appendix I all the formulae of the decay probabilities due to g-int. are collected, including those of spin 3/2 particles.

1. Introduction

If some regularity and universality are found in the transmutation processes between so many particles known so far, we may say that they might reflect some fundamental structures which are hidden behind the complicated interactions between them. The most suitable way of considering the various elementary particles has not yet been settled. There may be some attractive viewpoints which interpret many particles as the composite states of a small number of elementary particles. Here, as one of the possible phenomenological approaches which try to investigate the structure of the interactions, we want to continue the standpoint [1]) which classifies many particles into several families and regards the individual couplings as a result of certain kinds of universal interactions [2 †, 3]) between these families. As regards the classification of families of elementary particles, it seems to us reasonable to make the following choice, taking into account their statistics, conservation of heavy particles and the orders of the strength of their couplings:

(I) Nucleon family (N.F.)	fermions	ψ
(II) Lepton family (L.F.)		
(III) Boson family (B.F.)	bosons	φ
(IV) Electromagnetic field		A_μ

We supplement the discussions of § 4 in ref. [1]) by adding the following remarks to the interactions between families which seem to have some universal character.

1.1. ELECTROMAGNETIC INTERACTION "$eJ_\mu A_\mu$" or "e-int."

In spite of the variations of form of the charge-current density J_μ according to the spin values of the charged particles, the value of the coupling constant is unique. The "$F_{\mu\nu}$" type interactions which also satisfy the requirement of gauge invariance seem to be derived from the combinations of "e-int." and the other interactions which will be mentioned below. In this sense, we may regard the

† Reference[3]) is cited as I in this paper.

coupling constant e as the attribute of the electromagnetic field which presents itself when A_μ couples with other fields.

From the analysis of high energy phenomena, a theory involving two coupling constants seems to be required. That is, we must have strong interactions comparable with pion-nucleon interaction for the copious production of hyperons and heavy mesons and quite weak interactions responsible for their rather long life-times. At present, strong interactions seem to exist only between N.F. and B.F. From the analogy of pion-nucleon interaction we assume here the form of strong baryon-meson interaction as follows (Yukawa type).

1.2. STRONG BOSON-FERMION INTERACTION $G''\bar{\psi}\psi\varphi''$ or G-int.

Every set of possible combinations of baryons and heavy mesons cannot always belong to this G-class. That is, they must be compatible with the longevity of known particles. Conservation of some new quantum numbers for fast reactions are postulated by several authors [4]). According to them, reactions which destroy the conservation of these quantities cannot be fast. In this paper we classify interactions of elementary character into "even" and "odd" interactions according to whether they conserve new quantum numbers or not. Fast reactions will be realised by the combinations of only "even" interactions but weak ones cannot occur without including at least one "odd" interaction. Here no attempts are made about the detailed structure of strong interactions. For the following discussions we only use the tentative assumptions that the order of magnitude of the coupling constant G in the strong baryon-meson interaction (G-int.) is comparable with that of the pion-nucleon interaction ($G^2/4\pi \approx 1$ for pseudoscalar coupling), and that the G-int. must be "even" interactions.

From the phenomenological analysis of slow reactions, we may mention the following classes of weak interactions which seem to have a certain kind of universal character.

1.3. WEAK FERMI INTERACTION $f''(\bar{\psi}\psi)(\bar{\psi}\psi)_{,,}$ or f-int.

It is well-known that among spin 1/2 particles (N, P, μ, e, ν) this type of interaction is able to explain many phenomena (β-decay, μ-capture by nuclei and $\mu \rightarrow$ e-decay) with an almost unique value of the coupling constant f [5]), though some selection rules are necessary if we want to let this interaction work universally between

these particles. With respect to the universality of f-int., the following points seem to be worth mentioning:

(*a*) *Are the coupling types of f-int. process independent or not?*
The β-decay coupling is found to be $f(\bar{\psi}_P O_J \psi_N)(\bar{\psi}_e O_J \psi_\nu)$ with $(\bar{\psi}_e O_J \psi_\nu) = S \mp T + pP$ where p may be equal to 1 or 0 [6]). Thus, it is interesting to see whether $\mu^- + P \to N + \nu$ can be explained with f-int. obtained by replacing the electron by a μ-meson in β-decay coupling. Also a decision respecting the type of μ-decay is to be hoped for †.

(*b*) *Can f-int. be extended to include other Fermi particles?*
f-int. gives for the decays $\varkappa \to \mu + \nu + \nu$ and $\varkappa \to e + \nu + \nu$ lifetimes of the order $\approx 10^{-9}$ sec [3, 7]) and this might provide possible interpretations of $K_{\mu 3} \to \mu + ?^0 + ?^0$ and $K_{e3} \to e + ?^0 + ?^0$. (See, however, § 4 and 5.) In I, the possible β-decay and μ-decay of hyperons due to f-int. were discussed in detail. For instance, $\Lambda^0 \to P + e^-(\mu^-) + \nu$ will be less frequent by a factor 1/50—1/300 than $\Lambda^0 \to P + \pi^-$ ††. Recently, a few examples [8]) are reported which are inconsistent with the two-body decays of hyperons but can be interpreted as the above alternative decay modes due to f-int. In the above discussion we assumed that both hyperons and $\varkappa$ have spin 1/2. If we find higher spin values for these fermions, the question whether f-int. may include them must be investigated.

(*c*) *Are neutrino processes characteristic for universal Fermi interaction?*
So far as we know, the neutrino seems always to take part in the universal Fermi interaction. That is, $\mu \to e + e + e$, $\mu + P \to P + e$, $\varkappa \to e(\mu) + e(\mu) + e(\mu)$ etc. seem to be forbidden. In analogy to the A_μ in e-int., can the neutrino be regarded as the source of f-int.? It also appears in other weak reactions such as $\pi \to \mu + \nu$, $K_\mu \to \mu + \nu$ and $K_3 \to \mu(\text{or } e) + \nu + \pi^0$ which may explain some part of the decays of $K_{\mu 3}$ and K_{e3} †††. It may not be, however, the source of all weak interactions because we have other slow interactions which are not neutrino porcesses such as hyperon decays

† As the coupling types of Fermi interaction depend on the ordering of four fermions, the most complete universality will be the combinations of covariants which have some kinds of symmetry properties independent of ordering and process.

†† As is the case with the β-decay, pseudoscalar interaction does not contribute much to the $\Lambda^0 \to P + e^-(\mu^-) + \nu$.

††† Most of the experimental results used in this paper are based on those reported at the Padua Conference etc. see ref. [9]).

and boson decays into lighter bosons. If we cease to regard the neutrino as the source of f-int., we may extend f-int. so as to include the interactions between four baryons. In this case, coupled with G-int., they can lead to many interesting weak reactions such as hyperon decays. (See 3.2.)

In I we pointed out that there may exist another type of weak interaction which seems to have some universal character comparable with the universal Fermi interaction. This is:

1.4. WEAK BOSON-FERMION INTERACTION $g''\bar{\psi}\psi\varphi''$ or g-int.

In I, assuming spin 0 for bosons and spin 1/2 for fermions, an approximately constant value of the coupling constant g is found to be consistent with various processes such as π-decay, K_μ-decay and hyperon decays. There, taking into account seriously the fact that $\pi \to e+\nu$ and possibly $K_\mu \to e+\nu$ are rare events, we adopted the derivative couplings and found a value of g of the order $(g^2/4\pi)m_\pi^2 \approx 3.6\times 10^{-15}$. In § 2 a more detailed analysis will be given of g-reactions and some speculations on the parity assignments will be made on the basis of multiple particle theory.

In addition, we also have slow reactions between bosons:

1.5. WEAK BOSON-BOSON INTERACTION

$$\theta^0 \to \pi^+ + \pi^-, \quad K \to \pi + \pi^0, \quad \tau \to \pi + \pi + \pi \text{ etc.}$$

In this paper we do not touch the elementarity of these interactions. As $\pi^0 \to 2\gamma$ can be interpreted by the virtual processes due to G and e-int. without introducing new direct interactions between π^0 and photon, the standpoint that these transitions can take place by the intermediary of the universal interactions 1.1—1.4 will be possible. Also the possible decay schemes of K_3-decay, $K_3 \to \mu(e)+\nu+\pi^0$, may find their explanation in the virtual G- and f-int. mechanism. These points will be discussed in § 4.

2. The Analysis of g-Reactions

At present, we have only little knowledge about the spins of unstable particles. Moreover, if we set up universal interactions between families of elementary particles, competing and unwanted reactions will sometimes take place. It is a rather difficult problem to decide whether these are forbidden by some selection rules or by the detailed forms of the interactions. Taking these considerations into account, we want here to re-investigate the order of the

coupling constant g, paying at first no attention to the unwanted processes. We also include the cases where the spin values of baryons and heavy mesons vary up to 3/2 and 1 respectively. For spin 3/2

TABLE I

Direct coupling ($g^2/4\pi = 2.06\times10^{-15}$ from $\tau(\pi\to\mu+\nu) = 2.5\times10^{-8}$ sec, $m_\pi = 273m_e$ and $m_\mu = 207m_e$).

Decay scheme	Assumed mass (m_e)	Life-time (10^{-10} sec)	Spin	Interaction
$\Lambda^0\to P+\pi^-$	$m_\Lambda = 2182$ ($Q=37$ MeV)	7.6×10^3 2.0×10	$\Lambda = \frac{1}{2}$ $\Lambda = \frac{1}{2}$	$\bar\psi\gamma_5\psi\varphi$ $\bar\psi\ \psi\varphi$
$\Sigma\to N+\pi$ } $\Sigma\to P+\pi^0$ }	$m_\Sigma = 2328$ ($Q=110$ MeV)	7.9×10^2 9.7	$\Sigma = \frac{1}{2}$ $\Sigma = \frac{1}{2}$	$\bar\psi\gamma_5\psi\varphi$ $\bar\psi\ \psi\varphi$
$\Xi\to\Lambda^0+\pi$	$m_\Xi = 2575$ ($Q=60$ MeV)	3.6×10^3 1.4×10	$\Xi=\Lambda=\frac{1}{2}$ $\Xi=\Lambda=\frac{1}{2}$	$\bar\psi\gamma_5\psi\varphi$ $\bar\psi\ \psi\varphi$
$\Xi\to N+\pi$	$m_\Xi = 2575$	3.0×10^2 7.2	$\Xi = \frac{1}{2}$ $\Xi = \frac{1}{2}$	$\bar\psi\gamma_5\psi\varphi$ $\bar\psi\ \psi\varphi$
$K_\mu\to\mu+\nu$	$m_K = 970$	1.4×10 1.1×10	K = 0 K = 1	$\bar\psi\gamma_5'\psi\varphi$ $\bar\psi\gamma_5'\gamma_\mu\psi\varphi_\mu$
$\varkappa\to\nu+\pi$	$m_\varkappa = 970$	3.0×10	$\varkappa = \frac{1}{2}$	$\bar\psi\gamma_5'\psi\varphi$

In direct couplings $\pi\to e+\nu$ and $K_\mu\to e+\nu$ are faster than $\pi\to\mu+\nu$ and $K_\mu\to\mu+\nu$ respectively by the available phase space volume.

$\gamma_5' = 1$ or γ_5 according to the relative parity of the two fermions.

particles, we use the Rarita-Schwinger [10]) formalism. All formulae are given in Appendix I. In Table I, we list the calculated decay life-times for direct couplings using the value of $g(g^2/4\pi = 2.06\times10^{-15})$ adjusted to the $\pi\to\mu+\nu$ decay life-time. In Table II the same quantities are listed for gradient couplings ($g^2/4\pi\,.\,m_\pi^2 = 3.6\times10^{-15}$ from $\pi\to\mu+\nu$). For hyperon decays we may have radiative corrections† due to G-int. between baryons or between baryons and π-meson. For $\Lambda^0\to P+\pi^-$, the example is shown in Fig. 1.

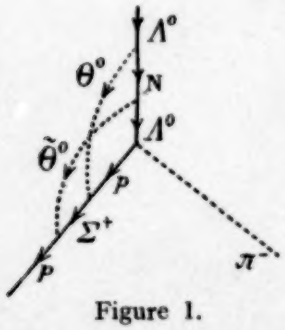

Figure 1.

† Note that such corrections will also occur for universal Fermi interaction which includes baryons. For β-decay the lowest order corrections due to π^0-meson are shown to be small, but in a good direction to lessen the small discrepancy between Gamow-Teller and Fermi coefficients [11]).

(G-int. are assumed here such as $\Lambda^0 \to N + \theta^0$, $\Sigma^+ \to P + \theta^0$ etc.). For $\pi \to \mu + \nu$ and $K_\mu \to \mu + \nu$ the main source of radiative corrections is expected to be due to e-int. Also, the present experiments on the decay life-times of unstable particles are not so definitely settled.

TABLE II

Gradient coupling $(g^2/4\pi \cdot m_\pi^2 = 3.6 \times 10^{-15})$

Decay scheme	Life-time (10^{-10} sec)	Spin	Interaction
$\Lambda^0 \to P + \pi^-$	2.0×10	$\Lambda = 1/2$	$\bar{\psi}\gamma_5\gamma_\mu\psi\partial_\mu\varphi$
	$\underline{7.1}$	$\Lambda = 1/2$	$\bar{\psi}\gamma_\mu \quad \psi\partial_\mu\varphi$
	2.6×10^4	$\Lambda = 3/2$	$\bar{\psi}_\mu\gamma_5 \quad \psi\partial_\mu\varphi$
	6.9×10	$\Lambda = 3/2$	$\bar{\psi}_\mu \quad \psi\partial_\mu\varphi$
$\Sigma \to N + \pi$ } $\Sigma \to P + \pi^0$ }	1.9	$\Sigma = 1/2$	$\bar{\psi}\gamma_5\gamma_\mu\psi\partial_\mu\varphi$
	$\underline{1.4}$	$\Sigma = 1/2$	$\bar{\psi}\gamma_\mu \quad \psi\partial_\mu\varphi$
	5.9×10	$\Sigma = 3/2$	$\bar{\psi}_\mu\gamma_5 \quad \psi\partial_\mu\varphi$
	7.3×10^{-1}	$\Sigma = 3/2$	$\bar{\psi}_\mu \quad \psi\partial_\mu\varphi$
$\Xi \to \Lambda^0 + \pi$	6.8	$\Xi = 1/2,\ \Lambda = 1/2$	$\bar{\psi}\gamma_5\gamma_\mu\psi\partial_\mu\varphi$
	$\underline{3.7}$	$\Xi = 1/2,\ \Lambda = 1/2$	$\bar{\psi}\gamma_\mu \quad \psi\partial_\mu\varphi$
	2.4×10^2	$\Xi = 3/2,\ \Lambda = 1/2$	$\bar{\psi}_\mu\gamma_5 \quad \psi\partial_\mu\varphi$
	6.4×10^{-2}	$\Xi = 3/2,\ \Lambda = 1/2$	$\bar{\psi}_\mu \quad \psi\partial_\mu\varphi$
	1.2×10^2	$\Xi = 1/2,\ \Lambda = 3/2$	$\bar{\psi}_\mu\gamma_5 \quad \psi\partial_\mu\varphi$
	3.2×10^{-2}	$\Xi = 1/2,\ \Lambda = 3/2$	$\bar{\psi}_\mu \quad \psi\partial_\mu\varphi$
$\Xi \to N + \pi$	6.5	$\Xi = 1/2$	$\bar{\psi}\gamma_5\gamma_\mu\psi\partial_\mu\varphi$
	5.6×10^{-1}	$\Xi = 1/2$	$\bar{\psi}\gamma_\mu \quad \psi\partial_\mu\varphi$
	1.1×10^2	$\Xi = 3/2$	$\bar{\psi}_\mu\gamma_5 \quad \psi\partial_\mu\varphi$
	2.7	$\Xi = 3/2$	$\bar{\psi}_\mu \quad \psi\partial_\mu\varphi$
$K_\mu \to \mu + \nu$	$\underline{1.4 \times 10}$	$K = 0$	$\bar{\psi}\gamma_5'\gamma_\mu \quad \psi\partial_\mu\varphi$
	$\underline{8.5}$	$K = 1$	$\bar{\psi}\gamma_5'\gamma_\mu\gamma_\nu\psi F_{\mu\nu}$
$\varkappa \to \nu + \pi$	1.4	$\varkappa = 1/2$	$\bar{\psi}\gamma_5'\gamma_\mu\psi\partial_\mu\varphi$
	1.9×10	$\varkappa = 3/2$	$\bar{\psi}_\mu\gamma_5' \quad \psi\partial_\mu\varphi$

For spin 1 K-meson $K_\mu \to e + \nu$ is faster than $K_\mu \to \mu + \nu$ by the available phase space volume.

The underlined values correspond to the case (δ) of 2.1.

From these situations the discrepancy of the order ten between theoretical values obtained without radiative corrections and present preliminary experimental life-times may be tolerable. Then Tables I and II seem to indicate the possibility that a unique value of the coupling constant g adjusted to the $\pi \to \mu + \nu$ decay is consistent with many reactions mentioned there, if we are allowed

to choose suitable sets of spin and parity for each particle. It may be noted that spin 3/2 baryons can take part only in the gradient couplings. It will not be easy to include these in the universal g-int., if we require the constancy of g for both hyperon decays and boson decays into leptons. (See 2.2.) It is perhaps premature to try to decide the type of universal g-int. The main conclusion of our analysis will be that in contrast to the strong G-int. ($G^2/4\pi \approx 1$) we may have a very weak g-int. ($g^2/4\pi \approx 10^{-15}$) and the value of g may be of a constant order of magnitude. From our tables we notice that the space inversion parity of baryons has remarkable effects. So it will be justified to add the following speculations.

2.1. PARITY ASSIGNMENTS DUE TO THE UNIVERSAL g-INT.

If we want to have more restricted interaction schemes, we may have some predictions on the parity of particles. Here, we give one simple example. For instance, let us adopt the simplest assumption that all members of N.F. and B.F. have spin 1/2 and 0 respectively, and take the multiple particle theory which requires that the interaction Lagrangian for g-int. must have the following form:

$$L_{\text{int}} = g\Sigma_a\Sigma_b\Sigma_c\bar{\psi}_a O_J \psi_b K_J \varphi_c + \text{c.c.} \tag{1}$$

We have four possibilities:

(α)	$O_J = \gamma_5$	$K_J = 1$
(β)	$O_J = \gamma_5\gamma_\mu$	$K_J = \partial_\mu$
(γ)	$O_J = 1$	$K_J = 1$
(δ)	$O_J = \gamma_\mu$	$K_J = \partial_\mu$.

From our tables we can see that (α) by itself is hardly consistent with experiments. The other three cases fit rather well and the best agreement with experiments seems to be given in case (δ). This seems to be the case, even if we do not confine ourselves to the spin assignments made above. The cases (β) and (δ) are adopted in I as W.B.-F.I. (weak boson-fermion interaction) †, and they also have the remarkable advantage that they can predict automatically that $\pi \to e+\nu$ and $K_\mu \to e+\nu$ are about 10^{-4} and 10^{-5} times less frequent than $\pi \to \mu+\nu$ and $K_\mu \to \mu+\nu$ respectively. For all other cases, the direct coupling (α) and (γ) and spin 1 K-meson, $\pi \to e+\nu$ and $K_\mu \to e+\nu$ are more frequent than $\pi \to \mu+\nu$

† The consistency of the co-existence of W.B.-F.I. with other universal interactions was discussed in I.

and $K_\mu \to \mu+\nu$ respectively and the introduction of some selection rule is inevitable.

Now let us make the convention that fermions of the type A ($\psi(\mathbf{x}) \to \gamma_4\psi(-\mathbf{x})$ for space inversion) and scalar mesons have intrinsic parity $+1$ and type B fermions ($\psi(\mathbf{x}) \to -\gamma_4\psi(-\mathbf{x})$) and pseudoscalar mesons have intrinsic parity -1. Let us define a nucleon to be of the A type. In our assumption of spin 0 for bosons, we must have at least two bosons of different parity in spite of their nearly equal masses; namely, K_1 which decays into two pions must be a scalar meson and K_2 which decays as $K_2 \to \pi+\pi+\pi$ must be pseudoscalar †.

(i) Case (β). (The coexistence of (α) will be allowed).

The experimental decay schemes require that in this case all known baryons must be of the A-type. Let us also take the straightforward generalisation of pion-nucleon interaction and assume the multiple particle theory †† in which "even" interaction is given by the "γ_5"-type.

$$L_{\text{int}} = G\Sigma_a\Sigma_b\Sigma_c\bar{\psi}_a\gamma_5(\gamma_5\gamma_\mu)\psi_b\varphi_c(\partial_\mu\varphi_c) + \text{c.c.} \qquad (2)$$

Then the K_1-meson (presumably θ-meson which has $\theta^0 \to \pi^+ + \pi^-$ and $\theta^+ \to \pi^+ + \pi^0$) cannot form strong interactions with known baryons. In this case only K_2-mesons are produced directly by π—N or N—N collisions. The abundance of K_1 production seems not to support this possibility. The possible way of escape is to assume the decay ††† $K_2 \to K_1 + 2\gamma$ which has comparable lifetime with other decay modes of K_2 and to assume quite rapid decay (in order not to destroy the coplanarity of K_1-decay) of the produced K_1. Then K_1-decay will look like the alternative decay of K_2. The mass difference * between K_2 and K_1 must be consistent with the rather long lifetime of the $K_2 \to K_1 + 2\gamma$ ($\tau \approx 10^{-8}-10^{-9}$ sec).

(ii) Case (γ) or (δ).

This case is more interesting because it leads to the parity assignments shown in Table III from known decay schemes. In

† Spin 1 for τ-meson seems to be excluded by experiments [12]).

†† Here we only consider the coexistence of G, e and g-int.

††† We need not depend on this mechanism, if unobservable heavier bosons (odd parity) which decay into K_1 with quite short lifetime, exist.

* Rough estimate gives the mass difference $\approx 25m_e$ for $\tau(K_2 \to K_1 + 2\gamma) \approx 10^{-8}$ sec [13]). See also ref. [22]).

this case $\Xi \to N+\pi$ is automatically forbidden contrary to case (i). This seems to have some experimental support †.

TABLE III

particles	N	Λ	Σ	Ξ	π	K_1	K_2
parity	+	−	−	+	−	+	−

The examples of their production mechanisms compatible with our parity assignments and associated production †† of new particles are as follows:

$\pi+N \to \Lambda(\Sigma)+K_1$ (not K_2) $\quad \pi+N \to N(\Xi)+K_1+K_1$
$\pi+N \to \Xi+K_2$ (not K_1) $\quad \pi+N \to N(\Xi)+K_2+K_2$
$N+N \to N+\Lambda(\Sigma)+K_1$ (not K_2) $\quad \pi+N \to \Lambda(\Sigma)+K_1+K_2$
$N+N \to \Lambda(\Sigma)+\Lambda(\Sigma)+K_2$ (not K_1) $\quad$ etc.
$N+N \to N+\Xi+K_2$ (not K_1)

The threshold energies are lower in K_1-production than in K_2-production and this will predict the asymmetry of K-meson production. Precise determinations of particles in associated production will provide tests of our assignments.

If we are allowed to include leptons in (1), $\pi \to \mu+\nu$ requires $K_2 \to \mu+\nu$ but $K_1 \to \mu+\nu$ is forbidden. Either K_1 or K_2 may decay into an electron and a neutrino if there is no other selection rule. Gradient coupling (β) and (δ) need not worry about this problem †††.

2.2. g-INT. ONLY RESPONSIBLE FOR THE WEAK INTERACTIONS BETWEEN N.F. AND B.F.

In § 1, we mentioned four classes of interactions which seem to have a certain universality. Among them we may deduce the g-int. from the combinations of G, e and f-int. This will be discussed in

† In our arguments, without introducing the linear combination of multiparticle interactions having both "γ_5" (α), (β) and "1" (γ), (δ), we cannot assign different parity for Λ and Σ[14]).

†† If we impose some special postulate such as charge independence on the G-int., some of these reactions will be forbidden and we must also specify the anti-particle K-meson.

††† If the decay mode $K \to e+\nu$ with the lifetime $\approx 10^{-8}-10^{-9}$ sec is found, it will suggest that the g-int. which causes this decay cannot belong to W.B.—F.I. In this case, the direct type g-int. (case (γ) or possibly case (α)) are favoured. Or, it will suggest that this K-meson has spin larger than 0.

the next section. As stated in 1.3(c), if neutrino porcesses are the feature of f-int. we cannot deduce hyperon decays by such G, e and f-intermediation, though we can for boson decays into leptons ($\pi \to \mu+\nu$, $K_\mu \to \mu+\nu$, $K_3 \to \mu(e)+\nu+\pi^0$ etc.). In this case the importance of the existence of the primary g-int. for the explanation of the hyperon-decays must be stressed. That is, the following standpoint will become possible: g-int. is responsible only for the weak interactions between N.F. and B.F. and weak interactions between B.F. and L.F. such as $\pi \to \mu+\nu$ and $K_\mu \to \mu+\nu$ may be derived (if we want economy in introducing interactions), for instance, by the intermediary of G, e and "neutrino process" f-int. This point will also be discussed in § 3. On this standpoint, from our tables, the possibility of getting nearly the same value of the coupling constant g seems to be increased, and the inclusion of spin 3/2 particles seems to become somewhat easier, because we need not adjust g to boson decays into leptons.

3. Possible Deduction of g-int. through other Possible Universal Interactions

If we want to reduce the number of universal interactions, the elementarity of g-int. has more room for discussions than other interactions, as was briefly indicated in I. So the possibility of explaining the g-int. in terms of the other three universal interactions is investigated in this section. As G-int. play a part in this mechanism, it is desirable to avoid, if possible, the use of perturbation treatment for G-int. As f-int. is very weak, the appearance of f-int. once is enough to make the reaction under consideration slow. This fact simplifies the following discussions. It will also be a good approximation to neglect the radiative corrections due to e-int. compared with G-int.

3.1. π-DECAY

π-decay through pion-nucleon interaction and one of the f-int., $\mu^- + P \to N + \nu$, is often discussed in the lowest order approximation of the pion-nucleon coupling. Here we discuss this problem in the light of universal interaction. Let us take the f-int. which consist of two baryons (a and b) and two leptons (μ, ν) or (e, ν), such as $f(\bar{\psi}_a O_J \psi_b)(\bar{\psi}_\mu O_J \psi_\nu)$ or $f(\bar{\psi}_a O'_J \psi_b)(\bar{\psi}_e O'_J \psi_\nu)$. First we make the following simplest assumption for them (assumption of process independence

of f-int.): these interactions are completely symmetric with regard to μ-meson and electron †.

This means that not only the possible combinations of baryons a, b which couple with (μ, ν) and (e, ν), but also the coupling types $(\bar{\psi}_\mu O_J \psi_\nu)$ and $(\bar{\psi}_e O'_J \psi_\nu)$ are the same for (μ, ν) and (e, ν). Then the main contributions to π-decay come from the summations of Feynman diagrams in which all vertices are G-int. except the one f-vertex where external lepton lines are drawn. (e-int. is neglected.)

Fig. 2.

Fig. 3.

As shown in Figs. 2 and 3, $\pi \to e+\nu$ is obtained by just replacing the μ-meson by an electron in the graphs describing $\pi \to \mu+\nu$ decay. As leptons under consideration have no G-int., we can include every radiative G-correction in our discussions. It is permissible to neglect e-correction at first. The matrix elements of these diagrams are given by

$$(\bar{\psi}_{(\mu \text{ or } e)} O_J \psi_\nu) M_J(\not{p}) \varphi_\pi(p) \tag{3}$$

where $\not{p}_\lambda$ is the energy-momentum four-vector of the external π-meson and $\varphi_\pi(p)$ is the wave function of the π-meson. The complicated contribution from various intermediate states is a function of $\not{p}_\lambda$ and the masses of the intermediate particles, and is denoted by M_J.

J shows the transformation property of M_J. From the structure of M_J we have the following results:

(i) $(\bar{\psi}_{(\mu \text{ or } e)} O_J \psi_\nu)$ = Scalar (S)
= Vector (V)
= Tensor (T)

For these lepton covariants we cannot construct $M_J(\not{p})$ which satisfies covariant requirements. That is, π-decay cannot occur by this mechanism.

(ii) $(\bar{\psi}_{(\mu \text{ or } e)} O_J \psi_\nu)$ = Pseudoscalar (P)

$M_J = M$, scalar function of $\not{p}_\lambda^2 = -m_\pi^2$ and intermediate particle masses. Then (3) is identical to the results given by the

† This assumption seems plausible because all the reactions except the π-decay modes appear to be symmetrical between μ-meson and electron. That is, (i) β-decay and $\mu^- + P \to N + \nu$, (ii) $\mu \to e+\nu+\nu$ decay (iii) $K_{\mu 3} \to \mu + ?^0 + ?^0$ and $K_{e3} \to e + ?^0 + ?^0$. As discussed in I, the discrepancy between $\pi \to \mu+\nu$ and $\pi \to e+\nu$ can be ascribed not to the asymmetry but to the structure of the interactions.

following primary interactions †:

$$M(\bar{\psi}_{(\mu \text{ or } e)}\gamma_5'\psi_\nu)\varphi_\pi + \text{c.c.} \tag{4}$$

In the ratio $R = W(\pi \to e+\nu)/W(\pi \to \mu+\nu)$ of the π-decay probabilities, M will disappear by the cancellation and we get $R \approx 5.5$ which is completely unacceptable.

(iii) $(\bar{\psi}_{(\mu \text{ or } e)}O_J\psi_\nu)$ = Pseudovector (PV)

M_J must be of the form $p_\lambda M'$ where M' is a scalar function of similar properties to M. Matrix elements (3) of this case are thus equivalent to those given by the following primary interactions adopted in I as W.B.-F.I.:

$$M'(\bar{\psi}_{(\mu \text{ or } e)}\gamma_5'\gamma_\lambda\psi_\nu)\partial_\lambda\varphi_\pi + \text{c.c.} \tag{5}$$

Thus R is proportional to $(m_e/m_\mu)^2$ and is $R \approx 1.3\times10^{-4}$ which seems not to be inconsistent with experiments so far [15]).

Here we must mention the role of e-int. In case (i), e-int. correction will be important because otherwise all cases of (i) are forbidden. The inclusion of internal photon lines which contain external μ-meson or electron will make all cases of (i) allowed, though it decreases greatly the probability at least by the factor $\alpha^2=(1/137)^2$. From the general argument [16]), however, under our assumption even if we take into account the radiative corrections due to e-int., we cannot expect the factor $\approx (m_e/m_\mu)^2$ in R except in the cases

$$(\bar{\psi}_\mu O_J\psi_\nu) = (\bar{\psi}_e O_J\psi_\nu) = PV \text{ or } V.$$

Thus PV or V-coupling (V has the drawback that the leading matrix elements must contain at least the factor α) must play essential parts in explaining the ratio R. The analysis of β-decay coupling, however, seems to indicate $S\mp T+pP$ ($p=1$ or 0) and to exclude PV and V [6]). Hence it is quite difficult to ascribe $\pi\to\mu+\nu$ to the above G, e and f mechanism if we stand firmly by our assumption of process independence of f-int.. Thus, so far as we adhere to the idea of eliminating the universal g-int., we are forced to choose at least one of the following alternatives, (a) and (b).

(a) *The value of the coupling constant f is unique, but the complete symmetry with regard to μ-meson and electron is invalid: "process dependent f-int."*

So far, our discussions are restricted to the ratio R of π-decay. As regards the estimation of the absolute values of the transition

† γ_s' is γ_s or 1, according to the relative parity of μ or e and ν.

probabilities (for instance, M and M' in (4) and (5) respectively) we have rather little knowledge obtained by the lowest order perturbation calculations (order Gf) including only nucleon fields with arbitrary cut-off or by making drastic simplifications of the summations over intermediate states. So we cannot compare with much confidence the transition probabilities calculated for different coupling types of f-vertex. From the knowledge of β-decay couplings, however, we can deduce the following general arguments for the possible interaction types if we want simple structure for f-int. First, we assume that possible combinations of baryons a, b appear in the same way in

$$f(\bar{\psi}_a O_J \psi_b)(\bar{\psi}_\mu O_J \psi_\nu) \text{ and } f(\bar{\psi}_a O'_J \psi_b)(\bar{\psi}_e O'_J \psi_\nu).$$

(i) $(\bar{\psi}_e O'_J \psi_\nu) = S \mp T$, $(\bar{\psi}_\mu O_J \psi_\nu)$ contains at least PV or PS. In this case $\pi \to e+\nu$ probabilities contain at least the factor α^2 compared with $\pi \to \mu+\nu$ which will be useful to explain the ratio R.
(ii) $(\bar{\psi}_e O'_J \psi_\nu) = S \mp T + P$, $(\bar{\psi}_\mu O_J \psi_\nu)$ contains at least PV. This is possible only if it turns out that $\pi \to \mu+\nu$ through PV can predominate over the same order process $\pi \to e+\nu$ through PS.

For every vertex, however, $\pi \to e+\nu+\gamma$ (also $\pi \to \mu+\nu+\gamma$) will occur. The first non-vanishing matrix elements are, in general, of the order Gfe. For the same type of f-vertex $\pi \to e+\nu+\gamma$ will be more frequent than $\pi \to \mu+\nu+\gamma$ by the available phase space volume. If experiments show great difference between $\pi \to \mu+\nu+\gamma$ and $\pi \to e+\nu+\gamma$ they will also suggest that $(\bar{\psi}_e O'_J \psi_\nu)$ and $(\bar{\psi}_\mu O_J \psi_\nu)$ have at least not completely the same coupling types. It must also be noted that for $(\bar{\psi}_e O'_J \psi_\nu) = S \mp T$ and V, $\pi \to e+\nu+\gamma$ will be more frequent than $\pi \to e+\nu$ because the transition probabilities contain the factor α for the former cases and at least α^2 for the latter. The situation is exactly the opposite for P and PV. Thus data on the frequency ratio $R' = W(\pi \to e+\nu+\gamma)/W(\pi \to e+\nu)$ would be quite interesting. If $R' < 1$, (i) will be unacceptable. On the other hand, if $R' > 1$, (i) will be the case. For (ii) it means that $\pi \to e+\nu$ through PS (the lowest order Gf) is less frequent than $\pi \to e+\nu+\gamma$ through $S \mp T$-vertex which are at least Gfe-processes. So if $R' \gg 1$, the case (ii) seems to be unacceptable. Thus the ratio R' will give an indirect test of β-couplings. The direct determination of the coupling type of $\mu^- + P \to N + \nu$ will also throw light on our standpoint. The more complicated situation which we do not consider in the above discussions, that the sets of

possible combinations of baryons a, b which couple with $(\mu.\ \nu)$ and $(e,\ \nu)$ are different, cannot be excluded. This is, however, at least not the case for nucleons. Namely, we have β-decay and $\mu^-+P\to N+\nu$. Some information will be given by investigating the possible alternative hyperon decays such as hyperon$\to$nucleon$+\mu+\nu$ and hyperon$\to$nucleon$+e+\nu$.

(*b*) *There are at least two different values of the coupling constant f.*

This is to imagine that the universal Fermi interaction has two coupling constants which will explain the β-decay and the $\pi\to\mu+\nu$ decay separately. One typical example is $f_1(S\mp T)+f_2PV$ which is process independent. From the β-decay experiments $f_1\geqq 10f_2$ is required [6,17]). f_2 is assumed to give the correct $\pi\to\mu+\nu$ and is also harmless to the rarity of the $\pi\to e+\nu$. The introduction of two coupling constants cannot assert its priority over that of g-coupling. This possibility can also be criticised by investigating the relative probabilities of $\pi\to\mu+\nu$ and $\pi\to e+\nu+\gamma$ as was discussed in I.

Thus it is shown that, in order to eliminate g-int. responsible for π-decay we are forced to add more complicated structures to f-int.. If (i) we agree to make such a relaxation as (*a*) or (*b*) in the universal and symmetrical character of f-int., and (ii), in addition, the values of M or M' in (4) or (5) turn out to be of the order g shown in § 2, we are in a position to deduce $\pi\to\mu+\nu$ without introducing direct g-interaction. If neither (i) or (ii) turns out to be the case, the introduction of primary g-int. is the simplest and the most natural procedure.

We cannot say much about the numerical computations of G, e and f-intermediation, so the possibility that such mechanism will not contribute much to the π-decay cannot be ruled out completely. For example, suppose that f-int. turn out to be the following "process independent" ones, $(\bar{\psi}_\mu O_J\psi_\nu)=(\bar{\psi}_e O'_J\psi_\nu)=S\mp T$. This assignment requires at least the factor α^2 for $\pi\to\mu(e)+\nu$ probabilities and it may be possible that G, e and f-mechanism is not responsible for $\pi\to\mu+\nu$. As discussed in § 2, for the g-int. of direct coupling type, we must add some selection rule which forbids $\pi\to e+\nu$-decay. If we adopt W.B.-F.I. as our g-int., they will predict $R=W(\pi\to e+\nu)/W(\pi\to\mu+\nu)=1.3\times10^{-4}$ and also $R''=W(\pi\to e+\nu+\gamma)/W(\pi\to\mu+\nu+\gamma)\approx 10^{-3}-10^{-4}$ because the factor $(m_e/m_\mu)^2$ also appears in the ratio R''. It seems, however, difficult to exclude the coexistence of W.B.-F.I. with G, e and f-

intermediation from the experiments on R'', because $\pi \to e(\mu) + \nu + \gamma$ are more likely to occur than $\pi \to e + \nu$ in the G, e and f-intermediation which is compatible with the rarity of the $\pi \to e + \nu$ and they will mask the ratio R'' which is derived only from the W.B.-F.I.

Remarks added in proof:

(1) If the upper limit 5×10^{-5}, given by Steinberger [15]) for $R = W(\pi \to e + \nu)/W(\pi \to \mu + \nu)$ is accurate enough to exclude the value 1.3×10^{-4} predicted by the cases 3.1(b) and W.B.-F.I., which treat electron and μ-meson symmetrically, then the cases 3.1(a) or W.B.-F.I. with selection rules such as 2.1(ii) will be favoured.

(2) If the G, e and f-int. mechanism is indeed responsible for π-decay, $\pi \to e + \nu + \gamma$ will have an appreciable frequency of occurrence compared with $\pi \to \mu + \nu$ and $\pi \to \mu + \nu + \gamma$. If this is not the case, it is unlike that f-int. is the main source of all weak interactions. However, the possibility of cancellation between various channels in the case $\pi \to e + \nu + \gamma$ cannot be completely excluded, though this phenomenon would be most unusual.

3.2. HYPERON DECAYS

Let us proceed to other g-reactions, the hyperon decays, assuming that f-int. have such properties as (a) or (b) discussed in 3.1 and are able to lead to $\pi \to \mu + \nu$ coupled with G and e-int.. In I, § 5, we already pointed out that hyperon decays might be deduced from G and f-int. †. As stated in 2.2, however, we must add a new feature to f-int. for this purpose, namely

(a) *f-int. must be extended to four baryons* (not neutrino processes).

To develop the similar procedure to 3.1 we make the following assumption and the following approximation. For brevity, Λ^0 is assumed to be spin 1/2 ††.

† Dallaporta[18]) also discussed this problem independently. As our arguments mainly concentrate on the ratio of competing decay processes, we may refer to his results as regards the rough order estimation of the absolute values of the transition probabilities.

†† This assumption seems plausible. Suppose some rules (conservation of some attributes etc.) exist between weak interactions. From $\pi \to \mu + \bar{\nu}$ and $\Lambda^0 \to P + \pi^-$, we may imagine that $(\mu + \bar{\nu})$ and $(P + \bar{\Lambda}^0)$ have some common attributes. Then it will be reasonable to expect from the standpoint of universal interactions that (μ, ν) and (P, Λ^0) are able to couple symmetrically with some sets of baryons in f-int.

Assumption: Let f-int. containing (μ, ν) and (P, Λ^0) be $(\bar{\psi}_a O_J \psi_b)(\bar{\psi}_\mu O_J \psi_\nu)$ and $(\bar{\psi}_a O_J \psi_b)(\bar{\psi}_\mathrm{P} O_J \psi_{\Lambda^0})$, where a and b are some sets of baryons. Among these f-int. there is complete symmetry between (μ, ν) and (P, Λ^0).

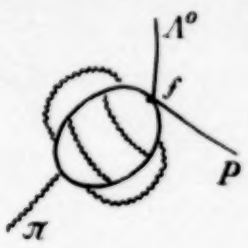

Fig. 4

Approximation: Λ^0 and P in f-int., as μ and ν, cannot interact strongly with other baryons and bosons.

Under the above assumption and approximation Λ^0-decay will be realised by the processes shown in Fig. 4 which have the same diagrams as π-decay in Fig. 2 as regards the intermediate states. We can take into account all radiative G-corrections which do not conflict with our "freezing" approximation. Then completely similar arguments to 3.1 are possible. Notations are the same as those used in 3.1.

(i) $(\bar{\psi}_\mu O_J \psi_\nu) = (\bar{\psi}_\mathrm{P} O_J \psi_{\Lambda^0}) = S$, V and T.

These are forbidden, but if we do not make the "freezing" approximation, some of them might be allowed by the inclusion of G^2-corrections (not e^2-corrections).

(ii) $(\bar{\psi}_\mu O_J \psi_\nu) = (\bar{\psi}_\mathrm{P} O_J \psi_{\Lambda^0}) = P$.

In place of (4), we have for $\Lambda^0 \to \mathrm{P}+\pi^-$

$$M(\bar{\psi}_\mathrm{P}\gamma_5'\psi_{\Lambda^0})\varphi_\pi + \text{c.c.} \tag{6}$$

(iii) $(\bar{\psi}_\mu O_J \psi_\nu) = (\bar{\psi}_\mathrm{P} O_J \psi_{\Lambda^0}) = PV$.

Instead of (5), we have

$$M'(\bar{\psi}_\mathrm{P}\gamma_5'\gamma_\lambda\psi_{\Lambda^0})\partial_\lambda\varphi_\pi + \text{c.c.} \tag{7}$$

Comparing (4) and (5) with (6) and (7) respectively, we can estimate the decay lifetimes of $\Lambda^0 \to \mathrm{P}+\pi^-$ from that of $\pi \to \mu+\nu$ without knowing the details of M and M'. From the results of the tables I and II, it is known that $O_J = 1$, $\gamma_5\gamma_\lambda$ and γ_λ can give the correct order of magnitude within the factor ≈ 10. The best fit is given in the case of γ_λ. This is interesting because it predicts the different space inversion parity for Λ^0 and P. Thus, if $\pi \to \mu+\nu$ can be explained by the G, e and f-mechanism, as discussed in 3.1, we can also have $\Lambda^0 \to \mathrm{P}+\pi^-$ in this mechanism. For Σ and Ξ-decay we can have the similar procedures taking also their spins to be 1/2 and we can also expect the workability of the same mechanism.

Finally, we must refer to the "freezing" approximation. The

main part of the G-corrections neglected just corresponds to the G-corrections to g-int. discussed in § 1 (e.g. Fig. 1). It is not certain whether such corrections which result when we do not make the "freezing" approximation, may be responsible for the discrepancy of the order of 2 to 10.

3.3. $K_\mu \to \mu+\nu$

Analogous mechanism is, of course, responsible for $K_\mu \to \mu+\nu$ as is shown in Fig. 5. We cannot, however, compare this process directly with $\pi \to \mu+\nu$. As K_μ has different attributes from those of the π-meson (e.g., different isobaric spin, strangeness), G-int. which appear in Fig. 5 cannot be identical to those in Fig. 2. Also the matrix elements of intermediate states depend not on p_μ of the π-meson but on p_μ of the K_μ-meson. By disregarding this dependence of the matrix elements on the intermediate particle masses and on the energy-momentum four vector of the initial meson, we get $\tau(K_\mu \to \mu+\nu) \approx 10^{-9}$ sec for K_μ, if we are successful in obtaining the correct $\pi \to \mu+\nu$ lifetime by G, e and g-intermediation. (See Table I and II). The fact that the masses of intermediate particles must be larger in the K_μ-decay case than in the $\pi \to \mu+\nu$ case will be of use in increasing the lifetime obtained above up to the order $\approx 10^{-8}$ sec †.

Fig. 5

To summarise our results it is shown that, in order to eliminate the elementary g-int., we must at least impose the more complicated structure 3.1(a) or 3.1(b) and the extension 3.2(a) to f-int. It is also shown that, if we admit these conditions, there still remains the possibility that g-reactions which are mentioned here can be derived from G, e and f-intermediation.

4. Competing K-Meson Decays due to Universal Interactions

We want to discuss here some aspects of K-meson-decays from the standpoint of universal interactions. Recently, the existence of the decay modes $K \to \pi+\pi^0$ has been confirmed by experiments. We also have other similar processes $\theta^0 \to \pi^+ + \pi^-$ and $\tau \to 3\pi$. These might be explained by some universal weak boson-boson interaction. We know, however, that $\pi^0 \to \gamma+\gamma$ may find a simple

† If K_μ-meson is pseudoscalar we can expect that the structure of f-int discussed in 3.1 will also predict the rarity of the $K_\mu \to e+\nu$ mode. So, if $K_\mu \to e+\nu$ is in fact rare, we are also tempted to assume $K_\mu = \tau$, as is the case in 2.1.

interpretation in virtual G and e process such as $\pi^0 \xrightarrow[G]{} P+\overline{P} \xrightarrow[e]{} \gamma+\gamma$. It is indeed a fast process and not inconsistent with the above mechanism. $K \to \pi+\pi^0$ is a rather slow reaction ($\tau \approx 10^{-8}-10^{-9}$ sec) and cannot be realised through "even" interaction (G-int.) only. Thus some "odd" interaction must take part in its decay mechanism. On the other hand, we have other charged boson decays, $K_{\mu 3} \to \mu+?^0+?^0$ and $K_{e3} \to e+?^0+?^0$ which seem to be no less frequent than $K \to \pi+\pi^0$. These K_3-decays might be interpreted as $\varkappa \to \mu$ (or e)$+\nu+\nu$ as stated in 1.3(b) We shall come back to this problem in the next section Here we interpret the K_3-decay as the boson decay $K \to \mu$ (or e)$+\nu+\pi^0$. As already noted in ref. [1]), from the standpoint of the universal interaction we may imagine at once the possible G, e and f-intermediation shown in Fig. 6 for its simple mechanism. Except for the f-vertex which contains leptons and is mainly responsible for the longevity of the K_3-meson, all other vertices must be G-int. There are some restrictions on this f-vertex. Consider the vertex which is obtained by replacing (μ, ν) in this f-vertex by a π-meson. If this new vertex is "even" interaction, $K \to \pi+\pi^0$ is able to occur as a fast inter-

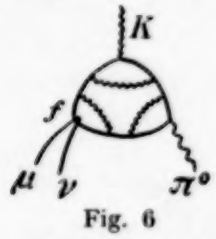

Fig. 6

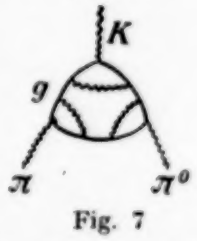

Fig. 7

action unless other selection rules forbid it. Thus the longevity of $K \to \mu(e)+\nu+\pi^0$ requires that in our mechanism two baryons which appear in the f-vertex must always be such possible combinations which lead to an "odd" vertex (such as g-int.) when leptons are replaced by π-meson. By making such replacement we want to investigate the possibility that $K \to \mu(e)+\nu+\pi^0$ is the alternative decay mode of the K-meson, which has a $K \to \pi+\pi^0$ mode. We call the new weak vertex comprising external π-meson a g-vertex in Fig. 7. First, we assume spin 0 for the K-meson. Then from $K \to \pi+\pi^0$, the K-meson under consideration must be K_1 (scalar) discussed in (2.1)†. Let the energy-momentum four-vector

† If we follow the multiple particle theory discussed in 2.1, we may have the following decay modes, if we have only two K-mesons, K_1 and K_2:

$K_1(\theta) \to \pi+\pi^0$ and $\mu(e)+\nu+\pi^0$ ($\mu+\nu$ is forbidden),

$K_2(\tau) \to \mu+\nu$ and $\pi+\pi+\pi$ ($\pi+\pi^0$ is forbidden).

of the K_1-meson, π^0-mesons in Figs. 6 and 7 be p_λ, q_λ and q'_λ respectively. We assume that both f- and g-int. have the baryon covariance of pseudoscalar type†. The transition matrix for each process is given by

$$gM(p,q)\varphi_K(p)\varphi^*_{\pi^0}(q)\varphi^*_\pi(p-q) \qquad \text{for } K\to\pi+\pi^0$$
$$fM'(p,q')\varphi_K(p)\varphi^*_{\pi^0}(q')(\bar\psi_\mu\gamma'_5\psi_\nu)(p-q') \quad \text{for } K\to\mu+\nu+\pi^0;$$

$M(p,q)$ is a scalar function of $p_\lambda^2=-m_K^2$, $q_\lambda^2=-m_\pi^2$, $(p\,.\,q)=-\frac{1}{2}m_K^2$ and the masses of particles which appear in the intermediate states. $M'(p,q')$ is obtained from $M(p,q)$ by replacing only $(p\,.\,q)$ by $(p\,.\,q')$. As $-m_K m_\pi \leqq (p\,.\,q')\leqq -1/2(m_K^2+m_\pi^2-m_\mu^2)$, the values of the ratio $(p\,.\,q')/(p\,.\,q)$ vary only from about 0.5 to 1. So if M does not depend radically†† on $(p\,.\,q)$, it will be a good approximation††† to take $(p\,.\,q)=(p\,.\,q')$ which means $M(p,q)=M'(p,q')$. The transition probabilities in the rest system of the K_1-meson are given by

$$1/\tau(K\to\pi+\pi^0)=2^{-3}(2\pi)^{-1}g^2M^2m_K^{-1}(1-4u^2)^{\frac{1}{2}} \tag{8}$$
$$1/\tau(K\to\mu+\nu+\pi^0)=2^{-5}(2\pi)^{-3}f^2M'^2m_K^3\{(1+u^2-2v^2)A- -\tfrac{2}{3}B-2v^4C\} \tag{9}$$

where

$$A=(1+u^2-v^2)w-4u^2\log\frac{1}{2u}(1+u^2-v^2+w),$$
$$B=w^3,$$
$$C=w-(1-u^2)\log\frac{1-2u^2-v^2-u^2v^2+u^4+(1-u^2)w}{2uv^2}$$
$$+(1+u^2)\log\frac{1}{2u}(1+u^2-v^2+w),$$
$$w=(1-2u^2-2v^2-2u^2v^2+u^4+v^4)^{\frac{1}{2}},$$
$$u=m_\pi/m_K, \qquad v=m_\mu/m_K.$$

Thus we get

$$R=\frac{\tau(K\to\pi+\pi^0)}{\tau(K\to\mu+\nu+\pi^0)}=2^{-2}(2\pi)^{-2}f^2g^{-2}M^2M'^{-2}m_K^4$$
$$\times(1-4u^2)^{-\frac{1}{2}}\{(1+u^2-2v^2)A-\tfrac{2}{3}B-2v^4C\}.$$

† If g-int. are derivative couplings, $\bar\psi_a\gamma_b'\gamma_\mu\psi_b\partial_\mu\varphi_\pi$ we can change them into the desired form, $(m_a\pm m_b)\,\bar\psi_a\gamma_b'\psi_b\varphi_\pi$, where upper and lower signs correspond to $\gamma_b'=\gamma_b$ and $\gamma_b'=1$ respectively.

†† This will be the case because p_λ^2 and q_λ^2 will also appear in the same way as $(p\,.\,q)$ and $(p\,.\,q')$.

††† It will be quite a good approximation if the emission of π^0-meson as a S-wave in the rest system of the K-meson is predominant.

Here we put $M = M'$ and assume, for instance, the same frequency ($R = 1$) for both reactions. Then taking Fermi constant $f = 1.4 \times 10^{-49}$ erg cm^3, we get $g^2/4\pi = 1.8 \times 10^{-15}$. This is strikingly in accordance with the value of g discussed in § 2 ($g^2/4\pi = 2.08 \times 10^{-15}$ for $\pi \to \mu + \nu$). Putting $v = 0$ in (9) we get the decay lifetime of $K \to e + \nu + \pi^0$. The ratio of the $K_3 \to \mu$ and $K_3 \to e$ decay probabilities is in this case $R = W(K \to e+\nu+\pi^0)/W(K \to \mu+\nu+\pi^0) \approx 2.3$. As discussed in 3.1($a$), there may be some doubts whether (e, ν) covariance of f-int. can be pseudoscalar. Though we cannot discuss with much confidence the frequency ratio in the case when it is derived from different types of (μ, ν) and (e, ν)-vertices, it will be true that we have comparable frequencies at least for the same types of f-vertices †. The experiments seem to show that the frequencies are comparable.

The energy distribution of the μ-meson in the above discussed $K \to \mu + \nu + \pi^0$ case is given by

$$P(E)dE = (m_K^2 + m_\mu^2 - m_\pi^2 - 2m_K E)^2 (m_K^2 + m_\mu^2 - 2m_K E)^{-2} (m_K E - m_\mu^2)(E^2 - m_\mu^2)^{\frac{1}{2}} dE.$$

Putting $m_\mu = m_e$ we can get the energy distribution of the electron. Thus in the above discussions we can show that by these categories of interactions (G, f and g-int.), the two possibilities of $K \to \pi + \pi^0$ and $K \to \mu(e) + \nu + \pi^0$ can be expected to have about equal probability. This will be preferable in the sense of economy of the number of K-mesons. We must also add the following arguments to the $K \to \pi + \pi^0$ decay. In the above discussions we only assign g-vertex to the outgoing π^+-meson. If we introduce weak g-int. comprising π^0-meson and K-meson (e.g., $\Lambda^0 \to N + \pi^0$, $N \to P + K_1^-$, etc.), g-int. may also appear in another vertex. On the whole, we may say that g-int., in effect, will appear in three external vertices with the same probability. This will make $K \to \pi + \pi^0$ more frequent than $K \to \mu(e) + \nu + \pi^0$. We do not compute the absolute values of the transition probabilities. Rough estimates will be seen in ref. [18]). Similar arguments can be done also for spin 1 (vector) K-meson, if the f-int. is still confined to the same type as the above. The transition matrix is given by

† In § 2 we raise the problem whether universal Fermi interactions, which contain leptons, are symmetrical between (μ, ν) and (e, ν). If the present interpretation of the mechanism of K_3-decay is true, we may expect some knowledge about the above problem from the analysis of K_3-decay.

$$gM_\lambda(p, q)\chi_\lambda(p)\varphi^*_{\pi^0}(q)\varphi^*_\pi(p-q) \qquad \text{for } K \to \pi+\pi^0$$

$$fM'_\lambda(p, q')\chi_\lambda(p)\varphi^*_{\pi^0}(q')(\bar{\psi}_\mu\gamma'_5\psi_\nu)(p-q') \quad \text{for } K \to \mu+\nu+\pi^0$$

where χ_λ is the wave function of the K-meson, M_λ is of the form $M_\lambda(p, q) = M_1(p, q)q_\lambda + M_2(p, q)p_\lambda$ where M_1 and M_2 are scalar functions with similar properties to M. It is, however, possible to drop M_2 by the equation $p_\lambda\chi_\lambda(p) = 0$. So we can develop the same procedure and also have the same conclusion (within the factor 10) as for the spin 0 K-meson. Our results may suggest that we can have similar conclusions for other types of f-int. and that they will be more reliable than the lowest order perturbation calculations which take the specific G-int. and an arbitrary cut-off procedure. We can also expect that the same mechanism will lead to anomalous θ^0-decay such as $\theta^0 \to \mu^\pm + \nu + \pi^\mp$ etc. Finally, we must add the following points. The discussions of this section are enough to show the workability of the universal g-int. They do not, however, allow any decision on the elementarity of g-int., because as was shown in § 3 there still remains the possibility that g-int. are derived from other universal ones, if we admit to add the complicated structure to f-int.

5. Concluding Remarks

In conclusion we want to summarise the important characters of the possible universal interactions and add some remarks on the possible existence of other sorts of interactions.

5.1. THE PRIMARITY OF UNIVERSAL INTERACTIONS

(*a*) *e-int.* The elementarity of the interaction is not in doubt. As regards the existence of the primary "$F_{\mu\nu}$" type universal interaction, the answer seems to be rather negative and we may be able to derive this type of interaction from the other universal interactions †. We need not assign this intrinsic interaction to an electron for the explanation of its anomalous magnetic moment. There seems also to be no positive evidence that the μ-meson has an appreciable anomalous magnetic moment ††.

† The predominant contributions to the "$F_{\mu\nu}$" type interactions are of the order eG^2 for N.F. and B.F. and e^3G^2 for I.F. in the present classification of interactions.

†† From the analysis of the frequency of bursts, Hirokawa *et al.*[19]) conclude that the anomaly of the μ-meson magnetic moment is less than 0.6 μ-meson Bohr magneton. See also ref. [23]).

(*b*) *G-int.* The strength of the coupling constant G does not allow one to deduce the G-int. from other universal interactions mentioned here. If it is true that G-int. are singled out from the interactions between N.F. and B.F. by the principle of charge independence and if nature is simple enough to assign spin 1/2 and 0 for N.F. and B.F. respectively, the following forms † of G-int. will be the most simple straightforward generalisation of the π-meson-nucleon interaction [14]):

$$L_{\text{int.}} = G\Sigma_a\Sigma_b\Sigma_c\bar{\psi}_a\gamma_5(\gamma_5\gamma_\mu)T_\alpha^{(ab)}\psi_b\varphi_c^\alpha(\partial_\mu\varphi_c^\alpha)$$

where $T^{(ab)}$ is a rectangular matrix in baryon isobaric spin space.

(*c*) *f-int.*

1. *The elementarity of f-int.* The primary importance of this interaction is rather high. In I it was pointed out that β-decay and μ^--capture by nuclei seem to be difficult to explain by G-int. and W.B.-F.I. The most convincing evidence is that, as leptons (μ, e and ν) known so far have no G-int., f-reactions such as $\mu \to e+\nu+\nu$ or possibly $\varkappa \to \mu(e)+\nu+\nu$ cannot be deduced from G, e and g-intermediation. (See also 5.2.) The practicability of the universality of f-int. will be helped if hyperon decays into leptons are confirmed, as was discussed in detail in I.

2. *The coupling types of f-int.* As the coupling types of f-int. depend on the ordering of the four fermions, we want to discuss here the f-int. which contain two baryons (a and b) and two leptons in the following forms: $(\bar{\psi}_a O_j \psi_b)(\bar{\psi}_\mu O_j \psi_\nu)$ and $(\bar{\psi}_a O'_j \psi_b)(\bar{\psi}_e O'_j \psi_\nu)$. The possible universal and symmetrical characters of f-int. are expected to show in this type of interaction. Two restrictions are imposed by the experiments on the f-int. of the type $(\bar{\psi}_a O'_j \psi_b)(\bar{\psi}_e O'_j \psi_\nu)$. These are

(α) β-decay coupling ($N \to p+e+\nu$) is required to be of the form $S\mp T+pP$ ($p = 0$ or 1);

† If we want to take the standpoint of unified field theory, the possibility of regarding the B.F. as the composite states of some elementary baryon and antibaryon which was first proposed by Fermi and Yang[20]) will be interesting. In this case strong Fermi interaction $G(\bar{\psi}\psi)(\bar{\psi}\psi)$ which consists of some elementary baryons may be a universal G-int. Our G-int. used in this paper may be regarded as the phenomenological one deduced from this interaction. Then there might remain the possibility that every reaction is explained by this strong Fermi interaction and weak f-int. which work among elementary fermions. Taking nucleons and Λ as the elementary baryons, S. Sakata proposes a theory in this direction (private communication).

(β) $\pi \to e+\nu$-decay which is inevitably caused by the combination of G, e-int. and $f(\bar{\psi}_a O'_j \psi_b)(\bar{\psi}_e O'_j \psi_\nu)$, must be consistent with the remarkable experimental fact

$$\tau(\pi \to e+\nu) > 10^{-4} \text{ sec.}$$

The experimental determination of the coupling types of the analogous f-int., $\mu^-+p \to N+\nu$, $\Lambda^0 \to p+e^-(\mu^-)+\nu$ and $\Sigma^\pm \to N+e^\pm(\mu^\pm)+\nu$ is desirable. The knowledge of the interaction types of $\mu \to e+\nu+\nu$ would also be enlightening. If, however, we want the simple structure of f-int. we can discuss the possibilities in more detail. At present, two standpoints (I and II) will have to be considered, because there seems to remain the possibility that f-int. is the main source of the weak interactions.

Standpoint I. "Process independent f-int." This is to require the process independent covariances. From (α), we must have the type $(\bar{\psi}_e O'_j \psi_\nu) = (\bar{\psi}_\mu O_j \psi_\nu) = S \mp T + pP$. As discussed in § 3, in this case we cannot explain the ratio $R = W(\pi \to e+\nu)/W(\pi \to \mu+\nu)$. Namely, we must introduce the g-int. for $\pi \to \mu+\nu$. The combination $S \mp T$ seems to be more favoured than $S \mp T+P$ because the existence of P will make it more difficult to obtain the necessary rarity of $\pi \to e+\nu$ †.

Standpoint II. "f-int. responsible also for g-reactions". From this standpoint we must first explain the decay lifetime and the ratio R of the π-decays by G, e and f-intermediation. If we adhere to the unique value of the coupling constant f, we are forced to have the following minimum relaxation of the simple structure of f-int. ††:

"The coupling types are process dependent". At present, the following types seem acceptable: (i) $(\bar{\psi}_e O'_j \psi_\nu) = S \mp T$, $(\psi_\mu O_j \psi_\nu)$ contains at least PV or P; (ii) $(\bar{\psi}_e O'_j \psi_\nu) = S \mp T+P$, $(\bar{\psi}_\mu O_j \psi_\nu)$ contains at least PV. The ratio $R' = W(\pi \to e+\nu+\gamma)/W(\pi \to e+\nu)$ will discriminate between these possibilities. That is, (i) will predict $R' > 1$ and (ii) $R' < 1$. The coupling types (i) or (ii) seem to work also for $K_\mu \to \mu+\nu$ and $K_3 \to \mu(e)+\nu+\pi^0$. If we further want to derive the hyperon decays without introducing the g-int., we must give up the possibility that f-int. are neutrino processes.

† In view of the sensitivity of the result to the cut-off procedure and in the absence of knowledge about the contributions from baryons other than nucleons and of a reliable strong coupling theory, we cannot form a conclusion for the absolute rate. We emphasise, however, that the conclusion derived from the ratio of π-decays is free from this defect.

†† In the sense that it introduces new freedoms it cannot assert priority over the introduction of g-int.

"*f*-int. must be extended between four baryons". It is interesting that from the hyperon decay schemes, if the "freezing" approximation made in 3.3(*b*) is good, we must also expect the existence of the *P* or *PV*-coupling for the covariances $(\bar{\psi}_P O_J \psi_\Lambda)$, $(\bar{\psi}_N O_J \psi_\Xi)$ and $(\psi_\Lambda O_J \psi_\Xi)$ etc., as is the case with $(\bar{\psi}_\mu O_J \psi_\nu)$ in (i) or (ii). This may suggest the existence of some symmetry † between *f*-int., even if it is forced to be process dependent from the standpoint II.

3. *Neutrino process hypothesis for f-int.* The extension of *f*-int. to four baryons may be a natural one. It is, however, to be remembered that the assumption that the neutrino is the source of *f*-int. does not contradict our experimental knowledge so far and is quite powerful in forbidding the unwanted reactions which contain leptons ††. For instance, it can forbid not only $\mu^- + P \to P + e^-$ and $\mu \to e+e+e$, but also $\Lambda^0 \to N + e^+(\mu^+) + e^-(\mu^-)$ and $\varkappa \to e(\mu) + e(\mu) + e(\mu)$ etc. Also in the reactions in which the *f*-int. may take part, it can exclude, for example, $K_3 \to \pi + e^+(\mu^+) + e^-(\mu^-)$, $\theta^0 \to e^+(\mu^+) + e^-(\mu^-)$ and $\theta^0 \to \pi^0 + \mu^+(e^+) + \mu^-(e^-)$ etc.

(*d*) *g-int.*

1. *The primarity of g-int.* The elimination of the primary *g*-int. is only possible when all the conditions imposed on the *f*-int. in 5.1(*c*)2 are satisfied, provided that in addition the correct $\pi \to \mu + \nu$ decay lifetime turns out to be given by *G*, *e* and *f*-intermediation. If any of these conditions is not fulfilled, the introduction of the primary *g*-int. is the most simple procedure. If the viewpoint of neutrino processes for *f*-int. is desired to be preserved, the primary *g*-int. is necessary at least for the weak interactions between N.F. and B.F. such as the hyperon decays.

2. *Coupling types of g-int.* The most promising interaction type which is consistent with an approximately constant value of the coupling constant for all *g*-reactions seems to be $g\bar{\psi}_a \gamma_\mu \psi_b \partial_\mu \varphi_c$ in the simplest spin assignment, spin 1/2 and 0 for fermions and bosons, respectively. This is interesting because it automatically predicts the rarity of the $\pi \to e + \nu$ and $K_\mu \to e + \nu$ mode and also requires different space inversion parity for particle-baryons.

If we want to take the standpoint that the *g*-int. is responsible only for the weak interactions between N.F. and B.F., then other coupling types will also be possible. If *G*-int. are singled out by the

† If experiments favour the rarity of $\Xi \not\to N + \pi$, we may imagine that the same mechanism of *f*-int. works also in this case as $\pi \to e + \nu$ and $K \to e + \nu$.

†† Sachs also noted this point. See ref.[4])

criterion of charge independence, these g-int. will be the charge dependent interactions between N.F. and B.F. and they might have some simple structure in isobaric spin space.

5.2. $\varkappa$-PARTICLE AND OTHER CLASSES OF INTERACTION

Let us suppose that at least some part of K_3-decay is explained by the decay modes $\varkappa \to \mu$ (or e)$+\nu+\nu$. Then we may expect some new aspects of the fermion families and their interactions. As $\varkappa$ cannot belong to N.F., it will be a higher mass level of leptons. As regards the copious production of K_3-meson comparable with other K-mesons, we may have two alternatives:

(α) $\varkappa$ is produced directly by high-energy nuclear interactions. That is, $\varkappa$ has strong interactions with N.F. and B.F.

(β) $\varkappa$ is only the decay product of short-lived massive parents. That is, $\varkappa$ has no strong interactions as other leptons.

In case (α) the simplest $\varkappa$-production mechanism, consistent with the stability of $\varkappa$ will be the pair production such as $\pi+\mathrm{N}\to\mathrm{N}+\varkappa+\varkappa$ or $\mathrm{N}+\mathrm{N}\to\mathrm{N}+\mathrm{N}+\varkappa+\varkappa$ which have higher threshold energies than the associated production of hyperons and heavy mesons. Such pair-wise production of $\varkappa$ must also be found by experiments. In this case, $\varkappa$ must have strong interactions (e.g., the extension of G-int. will be $G\bar{\psi}_\varkappa\psi_\varkappa\varphi_\pi$ etc. [22])). As the other members of L.F. have no G-int., the appearance of $\varkappa$ is very strange for our present classification of particles (see ref. [21])). In case (β) it is interesting to investigate the interactions which are responsible for the decay of the parents into $\varkappa$-meson. Let the parent B be a boson and the decay scheme be $\mathrm{B}\to\varkappa+\nu$. Assume the spins of B and $\varkappa$ to be 0 and 1/2 respectively. First, let us investigate the prediction of g-int.; the lifetime $\tau(\mathrm{B}\to\varkappa+\nu)$ of the decay $\mathrm{B}\to\varkappa+\nu$ is given by

$$\frac{\tau(\mathrm{B}\to\varkappa+\nu)}{\tau(\pi\to\mu+\nu)}=\frac{m_\pi\{1-(m_\mu/m_\pi)^2\}^2}{m_\mathrm{B}\{1-(m_\varkappa/m_\mathrm{B})^2\}^2}\quad\text{for direct coupling,}\qquad(10)$$

$$\frac{\tau(\mathrm{B}\to\varkappa+\nu)}{\tau(\pi\to\mu+\nu)}=\left(\frac{m_\mu}{m_\varkappa}\right)^2\frac{m_\pi\{1-(m_\mu/m_\pi)^2\}^2}{m_\mathrm{B}\{1-(m_\varkappa/m_\mathrm{B})^2\}^2}\quad\text{for gradient coupling.}\qquad(11)$$

The other spins and types of interactions do not change our conclusions essentially, so long as we keep the order of the value of g in § 2 ($g^2/4\pi\approx 10^{-15}$). The derivative coupling has the advantage

that it tends to make the mode $B \to \varkappa+\nu$ predominant over the competing modes $B \to \mu+\nu$ or especially over $B \to e+\nu$ without having recourse to some selection rules. Assuming, for example, $m_\mu/m_\pi = m_\varkappa/m_B$ and $m_\varkappa = 1000 m_e$, we get $\tau(B \to \varkappa+\nu) \approx 5\times10^{-9}$ sec and 2.5×10^{-10} sec for (10) and (11) respectively. Then this decay will be detectable in emulsions or cloud chambers. Assuming spin 1/2 for the parent B (baryon) and a decay scheme† $B \to$ nucleon$+\varkappa+\nu$ which will be caused by f-int., we also have a rather long lifetime $\approx 10^{-9}-10^{-10}$ sec for this decay. So far, it seems that we have no conclusive evidence of the existence of the parents of $K_{\mu3}$ and K_{e3}-particles. This may be an indication that g-int. and f-int. cannot be responsible for the decay of the parent into $\varkappa$-meson. Thus we may have the following two alternative possibilities:

(i) we have another interaction which is responsible for making the parent decay so fast and thus unobservable in emulsions ($\tau \leqq 10^{-14}$ sec);

(ii) the mass difference between B and $\varkappa$-meson is small enough to make the deflection of the track due to ν-emission ($B \to \varkappa+\nu$) small and make the decay unobservable.

In case (ii) it will be interesting to investigate whether the B-meson can be identified with the K-meson which has the other decay modes, e.g. $K \to \mu+\nu$ etc. In this case the branching ratio of K-decay will have the experimental tests

$$\frac{\tau(K \to \varkappa+\nu)}{\tau(K \to \mu+\nu)} = \frac{\{1-(m_\mu/m_K)^2\}^2}{\{1-(m_\varkappa/m_K)^2\}^2} \text{ for direct coupling,} \qquad (12)$$

$$\frac{\tau(K \to \varkappa+\nu)}{\tau(K \to \mu+\nu)} = \left(\frac{m_\mu}{m_\varkappa}\right)\frac{\{1-(m_\mu/m_K)^2\}^2}{\{1-(m_\varkappa/m_K)^2\}^2} \text{ for gradient coupling.} \qquad (13)$$

For instance, for $m_K = 970\, m_e$, $\Delta m = m_K - m_\varkappa = 40\, m_e$ (12) gives $\approx 1.4\times10^2$ and (13) ≈ 6.9, and for $\Delta m = 10\, m_e$ (12) gives $\approx 2.2\times10^3$ and (13) $\approx 1.1\times10^2$. The abundance of the K_3-meson seems to favour the gradient coupling. If (ii) is rejected by experiments, (i) will be the case. On the other hand, in our alternative (β) (in 5.2) $\varkappa$ cannot have strong G-int. as other leptons. Thus in case (i) we can imagine the existence of an interaction with an interaction constant g' which has a strength intermediate between G and g (e.g., $g'^2/4\pi \approx 10^{-7}-10^{-8}$). f-int. such as $\mu \to e+\nu+\nu$ cannot be

† Decay schemes such as $B \to \varkappa+\nu+\nu$ etc. will also require the parents of B.

intermediated by G, e and g-int. It will be, however, possible that this occurs by a g'^2 process through the unobservable bosons such as B. Of course, the introduction of such an interaction also requires new ordering principles †. From the standpoint of economy of the number of particles and interactions, it is hoped that K_3-decays will find their proper interpretations in the decay scheme $K \to \mu$ (or e)$+\nu+\pi^0$ as discussed in § 4 or $\varkappa$-production is the alternative decay mode of the K-meson which is slightly heavier than the $\varkappa$-meson.

It is a pleasure to thank Professor Rosenfeld for his interest in this work and for many stimulating and helpful discussions. We also wish to thank Professor T. Tati, Dr. S. Hori and Dr. S. Ogawa for their useful discussion. One of us (S.O.) is also indebted to the University of Manchester for the award of an ICI Fellowship.

Appendix I. Decay Probabilities due to g-int.

Formulae for the decay probabilities $1/\tau$ due to g-int. are listed including the cases where the spin values of fermions and bosons vary up to 3/2 and 1 respectively. For spin 3/2 particle, we use the Rarita-Schwinger formalism. The Rarita-Schwinger equation in the absence of interaction is

$$(\gamma_\mu \partial_\mu + m)\psi_\lambda = 0$$

where ψ_λ is a four-vector, each of whose components is a Dirac spinor. ψ_λ is required, in addition, to satisfy

$$\gamma_\lambda \psi_\lambda = 0.$$

AI.1. DIRECT COUPLING

(*a*) spin: $a = \frac{1}{2}$, $b = \frac{1}{2}$, $c = 0$
interaction: $H' = g\bar{\psi}(m_a)\gamma_5'\psi(m_b)\varphi(m_c) + \text{c.c.}$

decay scheme $a \to b+c$:

$$\frac{1}{\tau} = \left(\frac{g^2}{4\pi}\right)\left(\frac{m_a}{4}\right)\left\{\frac{(m_a-\varepsilon m_b)^2-m_c^2}{m_a^2}\right\}^{\frac{1}{2}}\left\{\frac{(m_a+\varepsilon m_b)^2-m_c^2}{m_a^2}\right\}^{3/2}$$

where $\varepsilon = 1$ or -1 according as $\gamma_5' = 1$ or γ_5.

† If f-int. can be intermediated by some interaction of such property as g'-int., there may be the possibility that this interaction is the main source of all weak interactions known so far. Recently an attempt in this direction has been made by S. Ogawa (to be published in Prog. Theor. Phys.).

decay scheme $c \to a+\bar{b}$:

$$\frac{1}{\tau}=\left(\frac{g^2}{4\pi}\right)\left(\frac{m_c}{2}\right)\left\{\frac{m_c^2-(m_a+\varepsilon m_b)^2}{m_c^2}\right\}\left\{\frac{m_c^4-2(m_a^2+m_b^2)m_c^2+(m_a^2-m_b^2)^2}{m_c^4}\right\}^{\frac{1}{2}}$$

(*b*) spin: $a=\frac{1}{2}, \quad b=\frac{1}{2}, \quad c=1$

interaction: $H'=g\bar{\psi}(m_a)\gamma_5'\gamma_\mu\psi(m_b)\varphi_\mu(m_c)+\text{c.c.}$

decay scheme $a \to b+c$:

$$\frac{1}{\tau}=\left(\frac{g^2}{4\pi}\right)\left(\frac{m_a^3}{4m_c^2}\right)\left\{\frac{(m_a+m_b)^2-m_c^2}{m_a^2}\right\}^{\frac{1}{2}}\left\{\frac{(m_a-m_b)^2-m_c^2}{m_a^2}\right\}^{\frac{1}{2}}$$
$$\times\left\{\frac{(m_a^2-m_b^2)^2-m_c^4+m_c^2(m_a^2+m_b^2-m_c^2-\varepsilon 6m_a m_b)}{m_a^4}\right\}$$

decay scheme $c \to a+\bar{b}$:

$$\frac{1}{\tau}=\left(\frac{g^2}{4\pi}\right)\left(\frac{m_c}{2}\right)\left\{\frac{m_c^2-(m_a+m_b)^2}{m_c^2}\right\}^{\frac{1}{2}}\left\{\frac{m_c^2-(m_a-m_b)^2}{m_c^2}\right\}^{\frac{1}{2}}$$
$$\times\left\{\frac{m_c^2-(m_a-\varepsilon m_b)^2}{m_c^2}\right\}\left[1-\frac{1}{3}\left\{\frac{m_c^2-(m_a+\varepsilon m_b)^2}{m_c^2}\right\}\right]$$

(*c*) spin: $a=\frac{3}{2}, \quad b=\frac{1}{2}, \quad c=0$

In this case the supplementary condition $\gamma_\lambda\psi_\lambda=0$ does not allow us to construct the interaction.

AI.2. DERIVATIVE COUPLING

(*a*) spin: $a=\frac{1}{2}, \quad b=\frac{1}{2}, \quad c=0$

interaction: $H'=g\bar{\psi}(m_a)\gamma_5'\gamma_\mu\psi(m_b)\partial_\mu\varphi(m_c)+\text{c.c.}$

decay scheme $a \to b+c$:

$$\frac{1}{\tau}=\frac{g^2}{4\pi}(m_a-\varepsilon m_b)^2\left(\frac{m_a}{4}\right)\left\{\frac{(m_a-\varepsilon m_b)^2-m_c^2}{m_a^2}\right\}^{\frac{1}{2}}\left\{\frac{(m_a+\varepsilon m_b)^2-m_c^2}{m_a^2}\right\}^{3/2}$$

decay scheme† $c \to a+\bar{b}$:

$$\frac{1}{\tau}=\left(\frac{g^2}{4\pi}\right)(m_a-\varepsilon m_b)^2\left(\frac{m_c}{2}\right)\left\{\frac{m_c^2-(m_a+\varepsilon m_b)^2}{m_c^2}\right\}\left\{\frac{m_c^4-2(m_a^2+m_b^2)m_c^2+(m_a^2-m_b^2)^2}{m_c^4}\right\}^{\frac{1}{2}}$$

(*b*) spin: $a=\frac{1}{2}, \quad b=\frac{1}{2}, \quad c=1$

interaction: $H'=g\bar{\psi}(m_a)\gamma_5'\gamma_\mu\gamma_\nu\psi(m_b)F_{\mu\nu}(m_c)+\text{c.c.}$

$$F_{\mu\nu}(m_c)=\partial_\mu\varphi_\nu(m_c)-\partial_\nu\varphi_\mu(m_c)$$

† The value given in I (§ 3 (6*b*)) contains misprints.

decay scheme $a \to b+c$:

$$\frac{1}{\tau} = \left(\frac{g^2}{4\pi}\right)\left(\frac{m_c^3}{4}\right)\left\{\frac{(m_a+m_b)^2-m_c^2}{m_a^2}\right\}^{1/2}\left\{\frac{(m_a-m_b)^2-m_c^2}{m_a^2}\right\}^{1/2} \times \left[\frac{2\{(m_a^2-m_b^2)^2-m_c^4\}-m_c^2(m_a^2+m_b^2-m_c^2+\varepsilon 6 m_a m_b)}{m_a^4}\right]$$

decay scheme $c \to a+\bar{b}$:

$$\frac{1}{\tau} = \left(\frac{g^2}{4\pi}\right)\left(\frac{m_c^3}{2}\right)\left\{\frac{m_c^2-(m_a+m_b)^2}{m_c^2}\right\}^{1/2}\left\{\frac{m_c^2-(m_a-m_b)^2}{m_c^2}\right\}^{1/2} \times \left\{\frac{m_c^2-(m_a-\varepsilon m_b)^2}{m_c^2}\right\}\left[1-\frac{2}{3}\left\{\frac{m_c^2-(m_a+\varepsilon m_b)^2}{m_c^2}\right\}\right]$$

(c) spin: $a=\frac{3}{2}$, $b=\frac{1}{2}$, $c=0$
interaction: $H' = g\bar{\psi}_\mu(m_a)\gamma_5'\psi(m_b)\partial_\mu\varphi(m_c)+\text{c.c.}$

decay scheme $a \to b+c$:

$$\frac{1}{\tau} = \left(\frac{g^2}{4\pi}\right)\left(\frac{m_a^3}{48}\right)\left\{\frac{(m_a+\varepsilon m_b)^2-m_c^2}{m_a^2}\right\}^{5/2}\left\{\frac{(m_a-\varepsilon m_b)^2-m_c^2}{m_a^2}\right\}^{3/2}$$

decay scheme $c \to a+\bar{b}$:

$$\frac{1}{\tau} = \left(\frac{g^2}{4\pi}\right)\left(\frac{m_c^5}{12m_a^2}\right)\left\{\frac{m_c^2-(m_a+\varepsilon m_b)^2}{m_c^2}\right\}\left\{\frac{m_c^4-2(m_a^2+m_b^2)m_c^2+(m_a^2-m_b^2)^2}{m_c^4}\right\}^{3/2}$$

Appendix II. Selection Rules on the Decay Modes $K \to \mu(e)+\nu$ and $K \to \mu(e)+\nu+\pi^0$

Here we summarise the selection rules which play a role if we take the viewpoint that the above decays occur through such mechanism as shown in Figs. 5 and 6. They become invalid only if we insert radiative corrections to the external lepton lines. As leptons have no strong interactions, the most important correction is due to e-int. which still decreases the probability by at least the factor $\alpha^2 = (1/137)^2$. These selection rules follow from the covariance requirement.

TABLE IV

$K \to \mu(e) + \nu$

f-int / K-meson	*S*	*V*	*T*	*PV*	*P*
Scalar	A	A	F	F	F
Pseudoscalar	F	F	F	A	A
Vector	F	A	A	F	F
Pseudovector	F	F	A	A	F

TABLE V

$K \to \mu(e) + \nu + \pi^0$

f-int. / K-meson	*S*	*V*	*T*	*PV*	*P*
Scalar	F	F	A	A	A
Pseudoscalar	A	A	A	F	F
Vector	F	A	A	A	A
Pseudovector	A	A	A	A	F

f-int. $= (\psi_{(\mu \text{ or } e)} O_f \psi_\nu)$
F = forbidden
A = allowed

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THE CONTRIBUTION OF NEUTRONS TO THE BACKGROUND OF COUNTERS USED FOR ^{14}C AGE MEASUREMENTS

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Received 8 March 1956

Abstract: Evidence is given that neutrons produced by cosmic radiation produce an important fraction of the residual background of counters used for ^{14}C age measurements. Neutron monitoring is essential in order to check irregular variations of the neutron flux, as for instance produced by solar flares. During the solar flare of Feb. 23 the average background for the whole night was 2.5 times normal.

If a counter is carefully shielded by a thickness of iron or lead to absorb gamma radiation and by an anticoincidence ring to suppress the effect of mesons, a residual background count remains. This is frequently due in part to radioactive contamination of the walls of the counter but for our new counters this contribution of radioactive contamination turned out to be very small. In the following discussion we will confine ourselves mainly to the largest counter (length 51.5 cm, diameter 7.0 cm) which had a background of about 5.5 c/m.

Whereas our first counter exhibited a very constant background[1]), the large counter fluctuated significantly, sometimes as much as 0.5 c/m. These fluctuations were correlated with changes in atmospheric pressure, being 6.0 per cent per cm Hg. This is much more than the variations reported by other stations (Münnich [2]), Fergusson [3])), and this is undoubtedly due to a lower contribution from radioactive contamination of the counter itself which is made of very pure electrolytic copper. The fluctuations in the meson effect were normal for the arrangement used, being 2 percent per cm Hg.

By analysing all our data on the background with different shields, counters, etc [4]). we concluded that of the known particles that might be responsible for the effect, neutrons were the most probable. This conclusion was supported by the following considerations and experiments:

1) The background was reduced from 5.50 to 5.10 c/m by a layer of 20 cm of paraffin wax, mixed with boric acid, placed on top of the normal layer of 38 cm of iron above the counter. A layer of 15 cm of lead in place of the paraffin wax gave no significant reduction of the background. Its scattering and absorption of neutrons is masked by the production of new neutrons (see Reich [5])). This same layer of lead placed *over* the wax yielded a reduction from 5.10 to 4.55 c/m. This reduction corresponds to the absorption of the neutron producing component of the cosmic radiation in the lead and the correspondingly smaller production of neutrons in the iron shield.

2) Neutrons have the correct barometric effect under the present conditions [5]). Within the limits of error the effect remained unchanged in all arrangements studied; all backgrounds quoted are referred to the "standard" barometric pressure of 76.6 cm Hg.

3) From the data on the number of neutrons at sea level and the scattering crosssections of the filling gas (CO_2) it follows that about one neutron per minute could be counted by direct recoil. Extrapolating the linear relation between gas pressure and background [4]) to zero pressure, the separate effects of the wall and the gas could be determined. The effect of the gas was 1.20 c/m; this is in accord with the estimate mentioned. Moreover, on filling the counter with "old" propane to 150 cm Hg pressure instead of carbon dioxide to 230 cm Hg, the background rose to 12.2 c/m. This increase is due to the larger scattering cross section of hydrogen for neutrons with energy below about 1 MeV, and to the larger energy of the recoils.

4) The background contains a larger proportion of low energy pulses than the ^{14}C spectrum. This undoubtedly corresponds to a large fraction of low energy recoils.

5) The insulation for the counter was provided by a cylinder of paraffin wax 1 cm thick. This cylinder can produce gamma radiation by neutron capture, and also produce beta activity in the copper wall. Indeed, the background was reduced from 5.10 c/m to 4.25 c/m by taking out this cylinder of paraffin wax. In order to trap the neutrons far enough from the counter and without the production of hard gamma's we will try the effect of blocks of paraffin wax, mixed with boric acid, at suitable places in the iron shield.

Our medium sized counter (length 37 cm, diameter 5.0 cm) with quartz walls had a background of 1.75 c/m. The fluctuations in this

counter were also 6 per cent per cm Hg. For the first small counter (length 16 cm, diameter 5.0 cm) the background was 2.8 c/m. Part of its background was obviously due to radioactive contamination of the walls, and fluctuations are only of the order of two per cent per cm Hg. Expressed in counts per minute, they are smaller than for the large copper counter in proportion to the size.

The most disturbing feature of the neutron effect is that it can show irregular and large fluctuations correlated with solar activity. We therefore now measure the neutron intensity in a BF_3 counter until the neutron effect is suppressed completely. Perhaps the only safe method is to place the counters under 5 to 10 metres of earth. The importance of the neutron monitoring is illustrated by the effects of the solar flare of Feb. 23. (cf. ref. [6]). The average count in the BF_3 counter during the night was 3.7 times normal. The extra counting rate in the ^{14}C counters was proportional to their size. In the medium sized counter the increase was 2.5 c/m; if this increase had not been taken into account, the age calculated for the sample would have been 5000 years instead of the correct 12000 years.

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NEWS ITEM

INTERNATIONAL CONGRESS ON THE FUNDAMENTAL CONSTANTS OF PHYSICS, TURIN, 6—11 SEPTEMBER 1956

This year the city of Turin will celebrate the centenary of Avogadro. On this occasion, the Italian Physical Society will organize, under the auspices of the International Union of Pure and Applied Physics and of the National Committee of Research, an International Congress of Physics which will take place in Turin during the period of the 6th to 11th September 1956, and will be centered on the fundamental constants of physics. It will accordingly be divided into various sections, among which the following are of interest for nuclear physicists:

Velocity of light: questions concerning very fast particles.
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People interested should contact Prof. R. Deaglio, General Secretary to the Congress, Istituto di Fisica dell'Università, via P. Giuria 1, Torino.

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Since the nomenclature in the field of Nuclear Physics is not yet quite unified and a number of illogical expressions are unfortunately still widely used, it is advisable to give the following indications about the terminology adopted in this journal.

(1) The word 'electron' applies to particles with an elementary charge of either sign. When it is desirable to emphasize the sign of the charge, the words 'positon' and 'negaton' are used. Thus the illogical phrase 'electron-positron pair' is replaced by 'electron pair', or, if required, by 'positon-negaton pair'.

(2) Nuclei (or nucleids, *not* 'nuclids') are conveniently divided according to the parity of the proton and neutron numbers Z, N, into four families as follows:

even mass nuclei { **even nuclei (Z even, N even)**; **odd nuclei (Z odd, N odd)**

odd mass nuclei { **odd proton nuclei (Z odd, N even)**; **odd neutron nuclei (Z even, N odd).**

(3) The expressions 'isotopic spin', 'isotopic quantum number' are replaced by 'isobaric spin', 'isobaric quantum number'.

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